Electromagnetic Theory and the Lorentz Transform

Sidney Bertram, Ph.D., Fellow IEEE

Abstract

The differential relationships of electromagnetics are derived as a direct extension of electrostatics by considering that charge fields are gas-like with a characteristic velocity *c*. Two kinds of transforms are involved: simple time transforms (no x terms) describe the generation of mutual energy at every point in the fields while the Lorentz transform describes how the energy developed in the fields sum to the effective energy at the charge centers. With propagation at a finite velocity in the fields, there is more time available for moving charges to interact with stationary charges, so their mutual energy is increased. Interacting charges are therefore subjected to induced forces when the energy changes as a charge accelerates. When charges are moving in conductors, their mutual energy is determined by their relative velocity, so they are subjected to ‘magnetic forces’ as described by the cross-product term in their energy. (The square terms are canceled by the energy associated with the ions left behind in the conductors by the moving charges.) The mutual energy formed by a particle field approaching a detector appears as a ring that converges on the detector along a cone such that its location on the cone at any time is related to the location of the moving particle by the Lorentz transform. If a moving particle decays, its ring, persisting for some time after the particle decays, gives the particle the appearance of an increased lifetime. The derivations are compatible with Einstein’s postulates.

Introduction

Einstein's 1905 paper[1] provided a basis for determining the effects of motion on the forces in a system. This was followed in 1912 by Leigh Page's[2] derivation of magnetic forces by transforming the electrostatic forces. As shown herein, some electrostatic forces relevant to the generation of the magnetic forces were not recognized at that time so they were not included in his analysis. Since it didn't alter the final result, their omission from the derivation was not recognized but it hid the physics behind the generation of the magnetic forces. With the background of Einstein's introductory statement in his famous 1905 paper that Maxwell's electrodynamics had a problem and needed a major overhaul, the apparently unexplainable origin of magnetic forces confirmed the idea that our universe isn't Newtonian. The judgement was premature; by transforming the mutual energy developed in the fields, instead of the forces, the effects associated with moving charges follow in a classical derivation that shows the missing forces.

The early work in electricity and magnetism was based on the belief that the electric and magnetic forces acted on imagined particles at the center of the charge fields. The derivations given herein are based on the premise that the interaction between charges takes place in their fields and involves a random motion, as in gases, at the velocity of light with respect to the center of each charge. Feynman[3] used a similar concept in Quantum Electrodynamics, but the author considers that Feynman's "photon gas" has all the properties of the charge fields, including their energy density ($\frac{1}{2}\varepsilon_0 E^2$) and associated mass. Only elementary classical mechanical concepts are used in the theory, so the derivations provide an intuitive basis for further study.

The following describes two quite different mechanisms involved in charge interactions: 1. The interactions between two charges takes place everywhere in the fields where they result in the development of the mutual energy density; and 2. The Lorentz transform describes the summing of these contributions to yield the mutual energy that is effective at the charge centers.

Local interactions – the ‘Magnetic’ forces

The following is intended to provide a theoretical foundation for the classical theory that evolved from experiments with constant or slowly changing currents in closed circuits. The related fields are therefore essentially stationary so propagation delays are not considered.

The Transverse Motional Mutual Energy

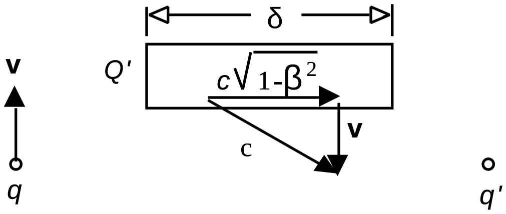

Fig. 1 Charge elements remain in *Q'* longer if *q* is in motion.

It is assumed that the charge fields are made up of exceedingly tiny elements that have an effective velocity *c* equal to the velocity of light.

Fig. 1 shows charge *q* moving with velocity **v** transversely with respect to the line to a stationary charge *q'*. A small cylindrical volume, *Q'*, of axial length δ in the direction of the electrostatic force on *q'* is fixed in the field of *q'*. It is assumed that the elements of the field of *q* that pass completely through *Q'* contribute to the effective mutual energy with *q'* in proportional to the time they spend in it.

With *q* stationary, those elements that move directly through *Q'* would be in it for a time $t=\delta/c$. With *q* moving transversely with velocity **v** only the elements associated with it that have a velocity component –**v** with respect to *q* would pass completely through *Q'*; they would have an effective velocity through *Q'* of $c\sqrt{1-\beta^2}$, where $\beta = v\ /c$, so the time they spend in it would be $t' = \delta\ / c\sqrt{1-\beta^2}$, or

$$t' = \frac{t}{\sqrt{1-\beta^2}} \tag{1}$$

It is assumed that this increase in interaction time causes a corresponding increase in the mutual energy developed in *Q'*. Since *Q'* could have been anywhere in the field, the integrated mutual energy seen by *q'* that is associated with the transverse motion of *q* is

$$M_\beta \approx \frac{q\,q'}{4\pi\varepsilon_0 r\sqrt{1-\beta^2}} \tag{2}$$

For the very slow charge velocities characteristic of the electrical circuits used in establishing the classical theory, (2) is approximated by

$$M_\beta \approx \frac{qq'}{4\pi\varepsilon_0 r}\left(1+\frac{\beta^2}{2}\right)$$

The one in the bracket yields the static mutual energy that is cancelled by the ions left behind in the conductors. Dropping it and, since the error involved is negligible, replacing the *nearly equals* with *equals*, yields the motional mutual energy associated with low velocity transverse relative motions,

$$M = \mu_0 \frac{qq'}{4\pi r}\left(\frac{v^2}{2}\right) \tag{3}$$

The constant $\mu_0 = 1/\varepsilon_0 c^2$ is called the *permeability* of space.

Establishing the forces between two charges when one is moving along the radial to the other and the second is moving transversely with respect to it, required to establish the cross-product force equation, requires a second motional mutual energy relationship as described later.

Induced forces

Since the mutual energy between two charges is a function of their relative velocity there are related forces as a charge accelerates. We say such forces are electrical since they do not involve the velocity of the second charge.

Fig. 2 shows charge q with an x-directed constant acceleration $\mathbf{a}$. It is assumed to have come in from a very large x, came to a stop at x=0 where it was a distance r along the perpendicular to the motion from a second charge q', and then moved back to the right. When x was near zero the velocity of q, $v = \sqrt{2ax}$, was essentially transverse to the radial to q', so (3) applies. Thus, for very small x, the mutual energy with the x-directed acceleration a of q in the field of q', is

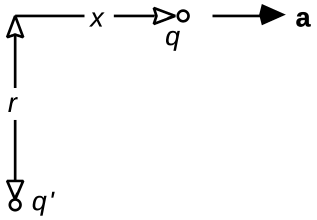

Fig. 2 Charge q, *a*ccelerating to the right near x=0, has velocity and acceleration nearly transverse to line to q'.

$$M_a \approx \mu_0\left(\frac{qq'}{4\pi r}\right)ax \tag{4}$$

Since M_a increases linearly with x there must be a positive x directed force acting on q and a reaction or *induced* force on q' of

$$\mathbf{f}_a \approx -\mu_0\left(\frac{qq'}{4\pi r}\right)\mathbf{a} \tag{5}$$

With interest centered on the forces on conductors, the charge q can be considered to be a charge element $\rho_l\,d\mathbf{l}$ on a differential conductor of length $d\mathbf{l}$, where ρ_1 is the charge per unit length of the conductor.

If the charges are moving with velocity $\mathbf{v}$ along the conductor, the differential current element is $\rho_l\mathbf{v}\,dl = i\,d\mathbf{l}$. Borrowing the classical notation, the induced force associated with transverse accelerations can be written in the form

$$\mathbf{f}' = -q'\frac{\partial\mathbf{A}}{\partial t} \qquad (6) \qquad \text{where} \qquad d\mathbf{A} \quad \mu_0\frac{i\,d\mathbf{l}}{4\pi r} \qquad (7)$$

The $d\mathbf{A}$ in (7) is the differential magnetic vector potential that integrates to yield the $\mathbf{A}$ of (6). Equations (6) and (7) are the differential equivalent of Faraday's law where motion of the conductors is not involved.

Note that if the acceleration is radial the x in (4) would be replaced by r so the mutual energy would be independent of r. Therefore there are no induced forces associated with radial motions.

The magnetic forces

The charges forming the currents in closed circuits are accompanied by the fixed ions they left behind in the conductors. Both sets of charges are involved in the development of magnetic forces.

Two transversely moving charges

Fig. 3 shows charges q and q' moving with parallel velocities $\mathbf{v}$, $\mathbf{v'}$ at a time when their separation is transverse to their velocities. Charges $-q$ and $-q'$, shown in parentheses, are ionized molecules left behind by the moving electrons; they are fixed in the conductors.

The motional mutual energy for this situation involves the charge pairs (moving q, moving q' with relative velocity [v-v']), (moving q stationary $-q'$ with relative velocity v) and (moving q', stationary $-q$ with relative velocity v'). The (moving q, stationary $-q$ with relative velocity v), and (moving q' stationary $-q'$ with relative velocity v') do not affect the external forces.

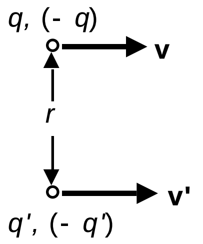

Figure 3 Charges q and q' moving with velocities $\mathbf{v}$, $\mathbf{v'}$ transverse to their separation are attracted by magnetic forces.

Using (3), the energy associated with the motion that leads to *magnetic* forces is

$$M = \frac{\mu_0}{8\pi r}\left[q q'\left(v - v'\right)^2 - q' q v^2 - q q' v'^2 \right] \tag{8}$$

This reduces to

$$M = -\mu_0 \frac{\left(q v\right)\left(q' v'\right)}{4\pi r} \tag{9}$$

While (9) suggests that the mutual energy effective in producing magnetic forces is only that between the moving charges, it is obvious that the ionized molecules left behind make a significant contribution to it – they cancel the energy associated with the squared velocities. Since the mutual energy is negative for charges of the same sign moving in the same direction, (9) leads to attractive forces between the paired charges of Fig. 3 of

$$f = \mu_0 \frac{\left(q v\right)\left(q' v'\right)}{4\pi r^2} \tag{10}$$

The force on q' is therefore upward.

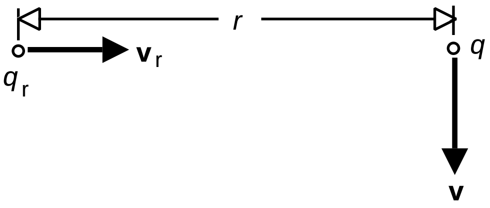

Fig. 4 Charges with orthogonal velocities have no mutual energy leading directly to magnetic forces.

Two charges having general motions

Fig. 4 shows charge q_r moving along the radial towards q with velocity $\mathbf{v}_r$ while charge q is moving transverse to the radial with velocity v. Since the velocities are orthogonal there is no mutual energy that leads directly to magnetic forces. However, forces are present as shown next.

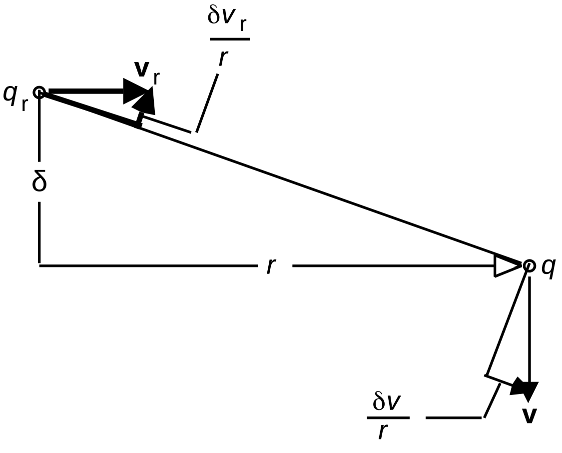

Fig. 5 . Offset orthogonal current elements have mutual energy leading to magnetic forces.

If q is displaced perpendicularly to the line to q (upwards) by a very small amount δ, exaggerated in Fig. 5, the situation is different; v_r then has a component $\delta v_r / r$ essentially parallel to v but with the opposite sense while v has a component $\delta v / r$

that is essentially parallel to v_r. Using (9), the contribution of the $\delta v_r / r$ term increases with δ, suggesting that there is a downward force on q_r – backwards from observations. However, as shown next, the contribution of $\delta v / r$ subtracts from this and results in the observed force.

Determination of the contribution of $\delta v / r$ to the motional mutual energy requires a determination of the time the motion associated with q' in Fig. 1 remains in the cylinder if q is moving along the radial to q'. In this case half the elements of q are moving directly towards q', so they have a relative velocity in the cylinder of $(c + v)$, while the other half are moving directly away from q' so their velocity relative to the cylinder is $(c - v)$. Assuming that their contributions are equal under static conditions, the mutual energy for motion along the radial is

$$M_{\beta r} = \frac{qq'}{8\pi\varepsilon_0 r}\left[\frac{c}{c+v} + \frac{c}{c-v}\right] = \mu_0 \frac{qq'}{4\pi r\left(1-\beta^2\right)}$$

Since, for low velocities, $1/(1-\beta^2) \approx 1+\beta^2$, this leads to a motional mutual energy for radial motion of

$$M_r = \mu_0 \frac{qq'}{4\pi r} v_r^2 \tag{11}$$

This is twice the value for transversely moving charges having the same velocity.

The analysis leading to (9) has a direct counterpart for the radial case with only a change of q' to q_r and v' to v_r. From Fig. 5 it can be seen that for the positive offset of q_r shown, the two radial components are in the same direction, so the mutual energy is negative. Since it is twice the positive contribution of the transverse component, their combined mutual energy is

$$M_{\delta r} = -\delta\, \mu_0 \frac{(q\, v)(q_r v_r)}{4\pi r} \tag{12}$$

This becomes more negative linearly with δ, so there is an upward force on q_r of

$$f_r = \mu_0 \frac{(q\, v)(q_r v_r)}{4\pi r^2} \tag{13}$$

This is the same as the force on the charge q' of Fig. 3. In terms of current elements the force is

$$f_r = \mu_0 \frac{(i\, dl)(i_r dl_r)}{4\pi r^2} \tag{14}$$

The cross-product

If a current element $i\,d\mathbf{l}$ is tangent at a general location on a circle centered on a current element $i'd\mathbf{l}'$, as shown in Fig. 6, resolution of $i'd\mathbf{l}'$ into components parallel to and perpendicular to the radial to $i\,d\mathbf{l}$ shows that there is a normal force on $i'd\mathbf{l}'$ equal to the value found above, so it is independent of where $i\,d\mathbf{l}$ is on the circle. It is therefore useful to use the direction of the circle, as described by the cross product of the direction of the current element $i\,d\mathbf{l}$ and the unit vector from $i\,d\mathbf{l}$ to $i'd\mathbf{l}'$ in describing the force. Using the classical notation, the differential expression for the normal force on $i'd\mathbf{l}'$ can be written as

$$d\mathbf{f} = i'd\mathbf{l}' \times \mathbf{B} \qquad (15\}$$

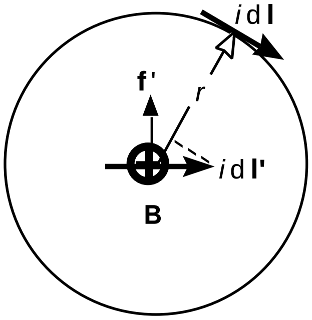

Fig. 6 Force $\mathbf{f}'$ on $i\,d\mathbf{l}'$ is independent of where $i\,d\mathbf{l}$ is on the circle centered on $i'd\mathbf{l}'$.

where $\mathbf{B}$, the "magnetic flux density" at the location of $i'd\mathbf{l}'$, points into the page. The differential magnetic flux density set up by $i\,d\mathbf{l}$ is

$$d\mathbf{B} = \frac{\mu_0}{4\pi r^2}\left(i\,d\mathbf{l} \times \mathbf{1}_r\right) \qquad (16)$$

Here $\mathbf{1}_r$ is a unit vector from $i\,d\mathbf{l}$ to $i'd\mathbf{l}'$.

The General Formula for Induced Forces

When currents change they produce induced forces as described by (6) and (7). Also, the factor $i'd\mathbf{l}'$ in (15) can be written as $q'\mathbf{v}'$ so a charge q moving in a magnetic field with velocity $\mathbf{v}$ experiences a magnetic force $q\mathbf{v}\times\mathbf{B}$. Combining this with the force given by (6) yields

$$\mathbf{f} = q\left[\mathbf{v}\times\mathbf{B} - \frac{\partial \mathbf{A}}{\partial t}\right] \qquad (17)$$

If the $\mathbf{v}$ in (17) is associated with the motion of a conductor then $\mathbf{v}\times\mathbf{B}$ is equivalent to an electric field that acts on the free charges in the conductor. Considering the bracket in (17) to be the effective $\mathbf{E}$, its line integral around any conducting loop yields Faraday's law that the induced voltage in a loop equals the rate at which the total magnetic flux in a loop is changing.

The Lorentz Transform

The Lorentz transform describes how the mutual energy components developed in the field coalesce as they move towards a detector.

Derivation of the Transform

The initial description is in terms of a central transverse component of a particle field that is moving towards a detector. At each point it passes, mutual energy components with the detector's field are developed across its extent. These radiate outwards as Huygens' sources in the field of the detector.

Fig. 7 shows the detector, *D*, a distance *x* from the origin at *O*. The dashed line represents a central component of the field of the moving charge that is moving with velocity **v** from *O* towards *D*. A Huygens' source *H*, on the line of motion, was formed earlier when the central component passed it. As the component moved on, the rays from *H* lying on a *ring formation cone* whose surface makes an angle $\theta = \arcsin\beta$ with the charge element, stayed with it and were reinforced by the emissions from off-axis Huygens' sources as the component passed. This produces a *particle ring* of growing strength and diameter, all associated with the time the element passed *H*. A continuum of these ring formation cones, each carrying information of the state of the particle as it passed their vertices, were formed as the component moved towards D.

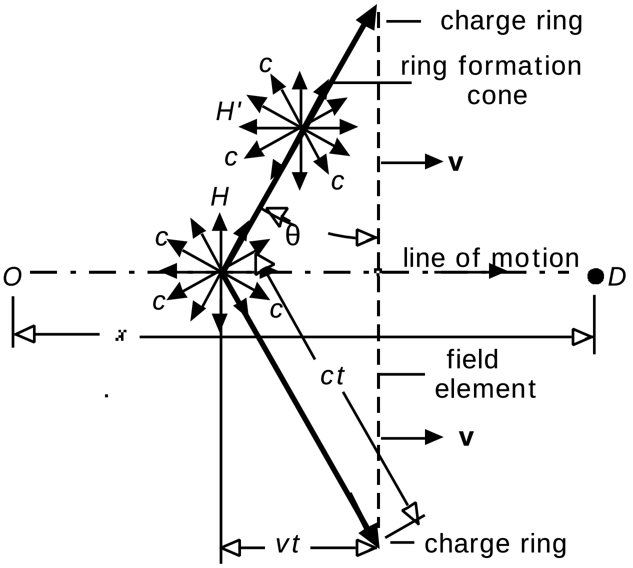

Fig. 7 Generation of charge ring

Fig. 8 shows the generation of the transform. The continuum of ring formation cones formed by the Huygens' sources as the central component moved towards *D* approach the detector along the cone orthogonal to them (three are shown); these rings serve as a surrogate for the moving charge in its interaction with the detector.

At a time *t*, when the wave element was at the position shown by the dashed line, the charge was at a distance *x-vt* from *D*, but its ring interacted with *D* as from its location on the orthogonal cone at a distance

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \tag{18}$$

from the detector. The ring represents the wave element at the time

$$t'' = t - \frac{d}{v} = t - \frac{\left(x - v\,t\right)}{v\sqrt{1-\beta^2}} \sin\theta \tan\theta \;\; = t - \frac{\left(x - vt\right)\beta^2}{v\left(1-\beta^2\right)}$$

when the wave element passed *H*. This reduces to

$$t'' = \frac{t - {}^{v}\!/_{c^2}\, x}{1-\beta^2} \tag{19}$$

Multiplying *t″* by $\sqrt{1-\beta^2}$ makes the velocity along the orthogonal cone equal the charge velocity and the time *t’* equal to that described by the Lorentz transform; that is

$$t' = \frac{t - {}^{v}\!/_{c^2}\, x}{\sqrt{1-\beta^2}} \tag{20}$$

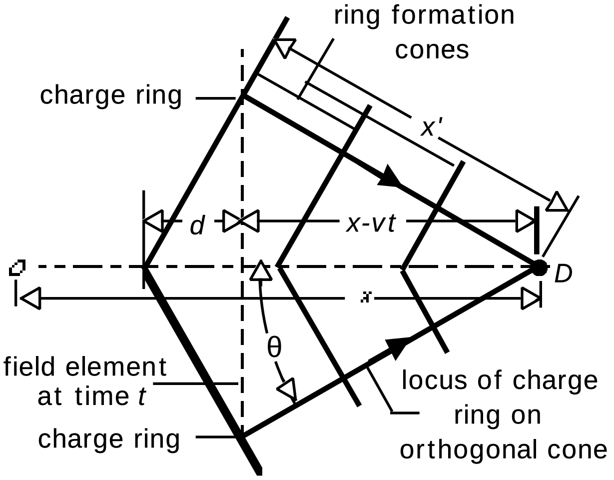

Fig. 8 Generation of the Lorentz Transform

Since all of the elements associated with the charge transform in the same manner, the ensemble of elements forming the charge is mirrored in a corresponding ensemble of circular elements that converge on the detector. The ensemble therefore serves as a surrogate for the particle in its reaction with the target, so (18) and (20) describe the motion of the particle as seen by the detector.

Conclusion

It has been shown that the assumption that the interactions between charges takes place in their fields provides the basis for a classical derivation of magnetic forces. The derivation of the Lorentz Transform suggests that we observe particles via surrogates formed in the local fields and that the observed “long lifetimes” of unstable particles is possible because the surrogates don’t vanish with the particles. The “slow clocks” of moving systems seem only a science fiction fantasy; there is no platform for the moving observer.

Einstein’s famous $E=mc^2$ probably applies to only the two-thirds of the energy associated with the transverse components of the electric field (the components that contribute to the magnetic fields). This explains part of the $E = \frac{3}{4} mc^2$ that Feynman wrote[4] “was discovered before relativity, and when Einstein and others realized it must always be $U = mc^2$, there was great confusion.” Since the longitudinal components don’t contribute to induced forces they probably don’t contribute to the effective mass either, so the experimental confirmation of Einstein’s equation is possible even though the energy involved is only two-thirds of the true energy. (The 3/4 instead of 3/2 seems to be due to counting the magnetic energy twice – the energy associated with the flow of the electric field is surely that associated with the magnetic field.)

As pointed out in the introduction, quantum electrodynamics[5] assumes that the interaction between particles takes place via photons that find their way between the particles. These photons can be identified with the fast moving elements assumed herein to make up the charge fields. As electrostatics teaches, the energy of a charge, and therefore also its mass, is in its field so, assuming that there is a velocity c associated with it, the reason for the mass/energy relationship can be seen. When a charge accelerates some of these elements break loose to form the photons of light and electromagnetic theory. Identifying the photons with these groups suggests that they have the same kind of mass the particles have, so there is no need for “General Relativity”.

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1 A. Einstein, “On the electrodynamics of moving bodies” **The Principle of Relativity**, Dover Publications, Inc. 1952

2. L. Page, A Derivation of the Fundamental Relations of Electrodynamics from those of Electrostatics,” *Am J Sci,* **34**,57-68 1912.

3. R. Feynman, **QED: The Strange Theory of Light and Matter,** Princeton University Press

4. R. Feynman et al, **Lectures on Physics**, Volume. II. (28-4)

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