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The Magnetic Force between Two Currents Explained Using Only Coulomb's Law

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A new explanation to the magnetic force between two conductors due to their respective currents is proposed. It is based upon the strict usage of Coulomb's original force law.

It is shown that this purely electrostatic law is capable of explaining also the magnetic force between currents. The reason is the inhomogeneous propagation of the electric field from different parts of continuously distributed moving charges, thereby causing a net difference between the field from the moving electrons and the immobile ions respectively in a conductor. Within the scope of this investigation it was also found that a D.C. voltage source must have inherited a direct current at the poles, opposite to the direction of the current through the circuit.

Using these concepts, experiments upon a set of Ampere's Bridge, performed by Moyssides and Pappas [J. Appl. Phys. 59, 19 (1986)] can be satisfactorily explained.

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I. Introduction

Since a long time ago it has been assumed that there exist both electric and magnetic fields. There seems to have existed strong historical reasons behind the development of separate field concepts.

Especially the fact that electric and magnetic phenomena have been observed separately for centuries, seems to have promoted this development.

As will be shown in this paper, a thorough analysis of the infinitesimal features of the propagation of the electric field departing from a continuously distributed charge, shows that there is a fundamental difference between the electric field caused by the electrons, moving through a conductor, and that of the immobile, positive ions. Thereby Coulomb's original force law is used, without any modification.

The difference thus appearing can account for the magnetic force between two conductors, carrying a current. How this happens is shown in the following text.

II. Coulomb's original force **law**

If questioning the established theory of electromagnetism, a reasonable beginning is to revert to Coulomb's law, the very foundation of electrostatics, a century before Maxwell connected magnetic and electric fields to each other through his famous equations.

II-l. Coulomb's **law**

Coulomb's law may in the case of distributed charges be written:

$$d^2\bar{F}/\,dx_1dx_2 = \rho_1\rho_2\bar{u}_R/4\pi\varepsilon_0 R^2 \qquad (1)$$

This relation is accepted today in the case of zero velocity. It shall now be shown that Coulomb's law does not need to be rejected in the case of moving charges, provided that the effects of retardation of action is correctly taken into account. Instead of the absolute charge densities, the virtual densities must be used.

11-2. **The virtual charge density of the sending point**

By the "virtual charge density" is meant the charge density an observer should see at a distant field point, when the charges at the sending point are moving. Assuming the configuration of Fig. 1, a field line which departs from the front of the charge element at time $t=$ 0 arrives at the field point, situated at the distance R, at time $t=R/c$.

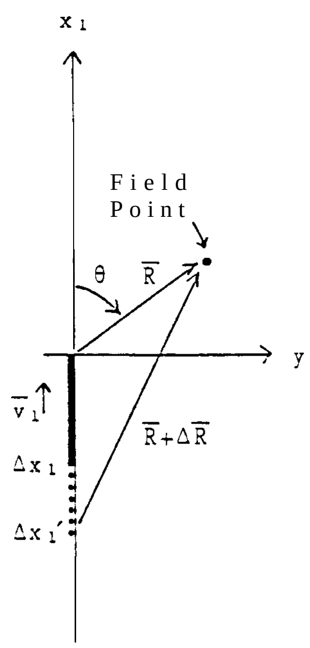

FIG. 1. Geometry in order to attain the virtual (observed) charge density at a distant field point.

A field line, leaving from the rear at $t = 0$ should arrive at $t = (R + \Delta R)/c$. This is of no interest. Instead, of interest is the field line which arrives at time $t =$ R/c at the field point. It must in this case be a question of a field line which left slightly earlier than at t=O from the rear, at time t = -AR/c. Assuming a total amount of charges

$$\Delta Q_1 = -\rho_1 \Delta x_1 \tag{2}$$

and the retarded time point of the rear, $t =$ -AR/c, corresponding to the retarded position $\Delta x_1'$, one may also write

$$\Delta Q_1 = -\rho_1' \Delta x_1'. \tag{3}$$

If the length of the charge element is virtually distorted, the charge density must change virtually too, provided the total amount of charges is unchanged. Letting the length of the charge element approach zero, one may write:

$$\rho_1' = \rho_1 \partial x_1 / \partial x_1'. \tag{4}$$

By geometrical reasons

$$\Delta x_1 - \text{Ax;} = v_1 \Delta R / c, \tag{5}$$

or in limes, after division by $\Delta x_1'$:

$$\partial x_1 / \partial x_1' = 1 + (v_1/c) \partial R / \partial x_1' \tag{6}$$

Further

$$x_1^2 + y^2 = R^2 \tag{7}$$

and

$$(x_1 - \Delta x_1')^2 + y^2 = (R + \Delta R)^2 \tag{8}$$

which gives

$$\partial R / \partial x_1' = -x_1 / R, \tag{9}$$

or, since $\bar{v}_1$ is parallel to $\bar{u}_{x_1}$,

$$\partial R / \partial x_1' = -(\bar{v}_1 \cdot \bar{R}) / v_1 R, \tag{10}$$

which together with Eq.(4) and Eq.(6) gives:

$$\rho_1' = \rho_1 (1 - (\bar{v}_1 . \bar{R}) / Rc). \tag{11}$$

11-3. Interaction with another current

In the preceding section it has been shown that at a distant field point, the charge density of a moving charge element is given a distorted picture.

Now, letting another charge element of velocity $\bar{v}_2$ be situated in this field point, shouldn't a field also meet a distorted charge density, by reason of symmetry? It will now be assumed that

$$R \gg |\Delta x_1|. \tag{12}$$

Then Fig. 2 gives an appropriate model of the arrivals of the field lines at charge element 2. The rear will be the first to receive action, the front slightly later, when it has moved from Δx_2 to $\Delta x_2'$, according to the relation

$$\text{Ax;} \; - \Delta x_2 = v_2 \Delta R/c, \tag{13}$$

or in limes

$$\partial x_2/\partial x_2' = 1 - (v_2/c)\partial R/\partial x_2'. \tag{14}$$

By geometrical reasons

$$x_2^2 + y^2 = R^2 \tag{15}$$

and

$$(\Delta x_2' - x_2)^2 \text{ t } y^2 = (\text{R} + \Delta R)^2, \tag{16}$$

which together gives

$$\partial R/\partial x_2' = -x_2/R, \tag{17}$$

or, since $\bar{v}_2$ is parallel to $\bar{u}_{x_2}, x_2$ negative,

$$\partial R/\partial x_2' = (\bar{v}_2 \cdot R)/v_2 R. \tag{18}$$

Then, in analogy with Eq. (11),

$$\rho_2' = \rho_2(1 - (\bar{v}_2 \cdot \bar{R})/Rc). \tag{19}$$

11-4. Electrostatic force due to currents

Taking into account the fact that the charge densities are virtually deformed according to Eqs. (11) and (19), when they are moving, Coulomb's law applied to two moving charge densities should read:

$$d^2\bar{F}/dx_1 dx_2 = \rho_1' \rho_2' \bar{u}_R / 4\pi\varepsilon_0 R^2. \tag{20}$$

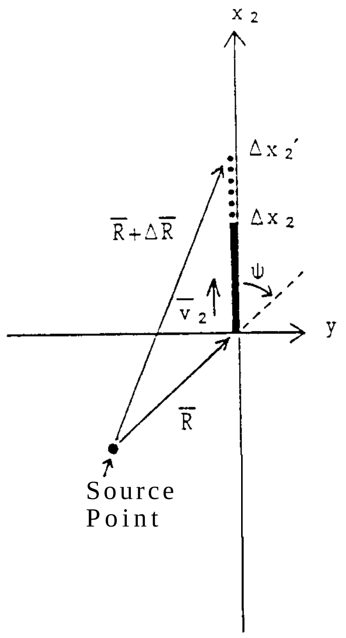

FIG. 2. Geometry in order to attain the virtual charge density upon which the electric field acts.

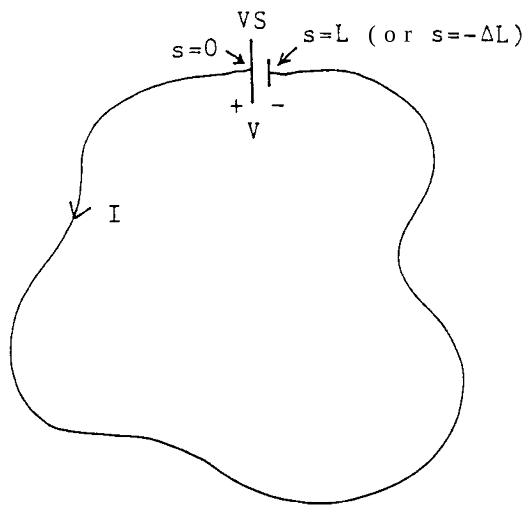

FIG. 3. Model of a closed circuit of arbitrary form, consisting of a capacitor battery and a conductor.

11-5. Application to the case of two straight conductors

Assuming two straight conductors, carrying a current, represented by moving charge densities, according to

$$I = \rho v, \tag{21}$$

there must be a zero net charge at both of them during normal circumstances.

If the charge density of the moving electrons is assumed to be ρ, then the charge density of the immovable ions must be $-\rho$. Because the velocity of the latter is zero, no virtual deformation takes place. In order to attain the Coulomb force from all the charges of one conductor upon all the charges of the other, one must add the virtual charge densities of the electrons and ions to each other on respective conductor. The net charge densities then becomes

$$\rho_1'' = -\rho_1(\bar{v}_1 \cdot \bar{R})/Rc, \tag{22}$$

$$\rho_2'' = -\rho_2(\bar{v}_2 \cdot \bar{R})/Rc. \tag{23}$$

ρ_1'' and ρ_2'' may now be inserted into Eq. (20) instead of ρ_1' and ρ_2'. Then one attains:

$$d^2\bar{F}/dx_1dx_2 = (-\rho_1\bar{v}_1 \cdot \bar{R}/Rc)(-\rho_2\bar{v}_2 \cdot \bar{R}/Rc)\bar{u}_R/4\pi\varepsilon_0 R^2 \tag{24}$$

Eq. (24) may be written

$$d^2\bar{F}/dx_1dx_2 = (\mu_0 I_1 I_2 \cos\theta \cos\psi)\bar{u}_R/4\pi R^2. \tag{25}$$

III. The magnetic force law

The magnetic force law claims that there exists a magnetic force between two currents according to

$$F_m = \int I_2 d\bar{s}_2 x B_2, \tag{26}$$

where the magnetic field $\bar{B}_2$ depends on I_1 according to Biot-Savart ' s law:

$$\bar{B} = (\mu_0/4\pi)\int (I_1 d\bar{s}_1 \text{ x } \bar{R})/R^3. \tag{27}$$

Thence.

$$\bar{F}_m = (\mu_0/4\pi)I_1 I_2 \int\int (d\bar{s}_2 \times (d\bar{s}_1 \; x \; \bar{R}))/R^3. \tag{28}$$

III-l. Comparison with Coulomb ' s law

The application of Coulomb ' s law to the case of two currents according to Eq. (25) and the Magnetic force law according to Eq. (28) shows many common features. Though not formally equal, both expressions are proportional to the respective currents and decrease proportionally to the square of the inverse distance. In order to decisively decide which equation is most qualified to describe the magnetic force accurately, measurement results from some relevant experiment must be accordingly investigated.

An interesting experimental situation, suitable for a test, is Ampere ' s bridge. An experiment series, performed by Moyssides and Pappas [1], gives the possibilities to compare the behavior of Coulomb ' s law with the Magnetic force law. This is done in a coming chapter.

IV. Voltage source theory

Any closed circuit, carrying a current, inevitably must include a voltage source. A deeper analysis of it will reveal some new, unexpected features. In order to succeed with the mathematical treatment of an arbitrary voltage source, most convenient is to treat the most simpel one first, a capacitor battery, which is not inherent with any chemical complications. Nonetheless this shows the most principal features of a D.C. source, if only the time constant is great enough. Here it is assumed to consist of a circuit of length L and a battery of length **AL, AL** $\ll L$.

In order to commit the analysis of the voltage source, the natural coordinate s is arbitrarily chosen. When a charge travels between the poles, a net work is done upon it fron the electric field. The time rate of this is the effect,

$$P = V \cdot I \tag{29}$$

According to the Law of conservation of energy, no net work can be done within a closed system. Therefore the same work most be done in the opposite direction.

This is on a principal level nothing else, than what an austronaut encounters in his spacecraft, when trying to move within the state of weightlessness.

The loss of one charge at one pole causes a drop in the voltage, hence a loss of force between the poles, and the charges, which remain there will be allowed to move according to this loss in some way or another.

Assume the displacement is As. This causes an ovarall drop in the voltage

$$V' = \frac{\Delta s}{L} V. \tag{30}$$

This voltage drop is connected with an effect P', which must obey

$$P' = -P \tag{31}$$

because of the law of conservation of energy. Hence, the current becomes

$$I' = \frac{-I \cdot L}{\text{As}}. \tag{32}$$

This is not yet good mathematics. With this form, the current must be definied as a spatial function. Convenient to this situation is the Heaviside function. Thus,

$$I'(s) = \frac{-IL}{\Delta s}(H(s - L) - H(s - L + \Delta s)). \tag{33}$$

This is convenient with the fact that there cannot be a current anywhere than where there is a voltage drop. Letting As approach zero, one easily becomes

$$\lim_{\Delta s \to 0}(I'(s)) = -I \cdot L \,.\, \delta(s - L) \tag{34}$$

V. Ampere's bridge

Experiments with a set of Ampere's bridge have been performed by Moyssides and Pappas [1]. Before describing their experimental set, it has to be mentioned that they make references to several other scientific articles with relevance to the subject [2–19]. Their set of Ampere's bridge consisted of a closed circuit, cut off at two parallel points according to Fig. 4 or 5. After these changes, the text should read as before. Basically it consisted of a closed circuit, cut off at two parallel points according to Fig. 4 or 5.

The points were electrically connected by mercury cups. As the current rose, an increased force was needed to keep the two branches together. The bridge was held parallel to the earth plane. The width of the conductor was varied between 1,9 and 3,0 mm, circular cross section. The current was varied from 60 to 220 A. No explanation to the behaviour of the force is given in the report [1]. An explanation based upon Ampere's law has been proposed by Wesley [2].

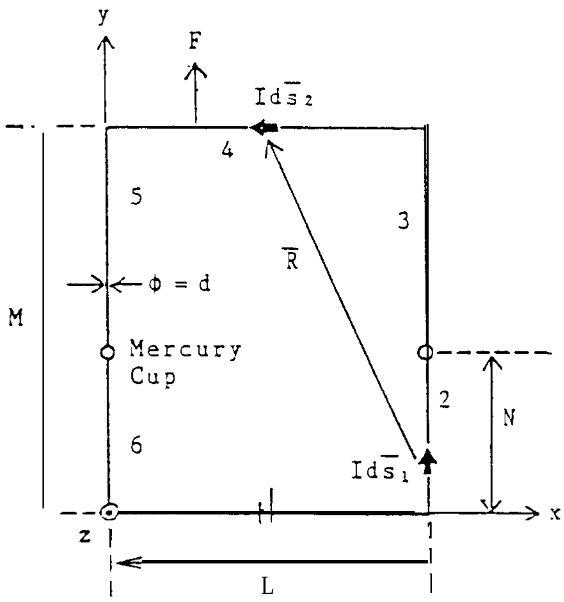

FIG. 4. Diagram of the experiment for the force on Ampere's bridge indicating coordinates, labelling and geometry. The case of straight ends.

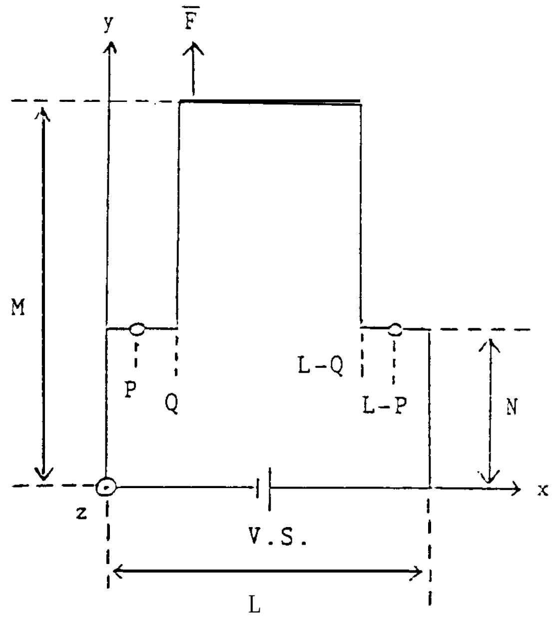

FIG. 5. The case of bent ends.

V-l. The electrostatic force according to Coulomb's law

The analysis consists of several steps. First Eq. (25) has to be used, thereby inserting

$$I_1 = I_2 = I. \tag{35}$$

The force needed to keep the bridge together is directed along the y axis. Hence, it is only needed to evaluate the y-component of the total force. This depends of the fundamental symmetry of the circuit, but can of course be questioned, too.

When interacting parts of the bridge are in close contact with each other, Eq. (25) must be modified. Current densities must replace the currents. Otherwise singularities will make the evaluation impossible. Thus, Eq. (25) transforms into:

$$d^6\bar{F}/(dx_1dx_2dy_1dy_2dz_1dz_2) = \frac{\mu_0}{4\pi}\frac{(\bar{J}_1\cdot\bar{R})(\bar{J}_2\cdot\bar{R})\bar{R}}{R^5}. \tag{36}$$

For simplicity, Eq. (25) may be written in a similar way:

$$d^2\bar{F} = \frac{\mu_0}{47r}\frac{I^2\bar{R}(d\bar{s}_1\cdot\bar{R})(d\bar{s}_2\cdot\bar{R})}{R^5}. \tag{37}$$

The total force upon Ampere s bridge may now be evaluated for the different cases, thereby using Eqs. (36) and (37) with respect to the different parts of the bridge. Also the dirac current, related to the voltage source, according to Eq. (34), must be taken into account.

V-2. Predictions of the force according to Coulomb's law

In the case of a bridge with straight ends, according to figure 4, $L = 0,48$ m, $M = 1,20$ m, N $= 0,43$ m.

In the case with the cross section 1,9 mm, the predicted force is 4,79 (gram weight/amp') $\cdot 10^{-5}$. Cross section 3,1 mm implies a force of 4,08 (same units).

In the case of a bridge with bent ends, according to figure 5, independently of the cross section, the force is predicted to be 3,47 (same units), for a set, where $P = 1$ cm, Q = 2 cm, L = 52 cm, M = 120 cm and N = 43 cm. For another set, with $P = 1$ cm, Q = 3 cm, L = 54 cm, M = 120 cm and N = 43 cm, the force predicted is 3,05 (the same units as above).

The dependence of the cross section in the case of straight ends depends of the volume integrals of Eq. (36), from which a net y component of the force along the y axis arise. In the case of bent ends, there is no y component of the force from current elements close to each other. The only forces come from line integrals.

V-3. Experimental results

The predicted values above will now be compared with measurement results according to Pappas and Moyssides.

Straight ends	Measurement	Prediction
Cross section 1,9 mm	11,8	4,79
Cross section 3,1 mm	9,6	4,08
Bent ends		
First case	7,04	3,47
Second case	6,06	3,05

V-4. Prediction according to the magnetic force law

Using Eq. (28) for the case of straight ends gives a predicted force of strength 0,25 (the same units), independently of the cross section. There is no such dependence, because there is no y component of the force from current elements close to each other, i.e. line integrals are suffient in this case. Integrals for the case of bent ends remain to be solved.

VI. Conclusions

From the investigation above it is clear that Coulomb's law can account for the principal behavior of the measured force. When the Magnetic Force law is used accordingly, it is not possible to see any correlation, but further investigations ought to be carried out,

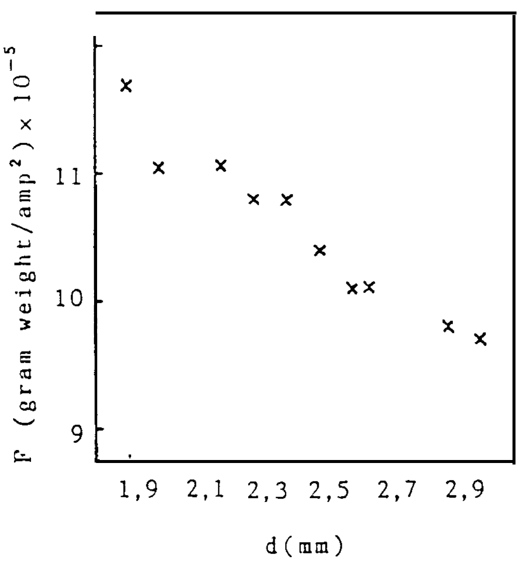

FIG. 6. Experimental points ot Moyssides and Pappas for the case of straight ends.

before judging the case finelly. From the investigation above it is very clear, that the magnetic force law fails completely to explain the force upon the bridge in the studied case. Coulomb's law succeeds at least qualitatively, even though the values are lower than the measured. However, if studying the quotas between the expected and measured values respectively, they are practically the same, 0,41-0,42 in the case of straight ends and 0,49-0,50 in the case of bent ends. Perhaps only a scale constant is needed. Further work must be done upon other configurations in order to analytically define this.

Appendix

Throughout the text MKS units are used. Most of the variables are defined in the text or by a figure. Some explanations are nevertheless inevitable.

$\bar{F}$	electrostatic force according to Coulomb's law
$\bar{F}_m$	magnetic force according to the magnetic force law
ρ	charge density
ρ'	virtual (observed) charge density
ρ''	net virtual charge density due to a conductor current
$\bar{u}$	(with index) unit vector
VS	voltage source
w	width of the conductor of Ampere's bridge
t	laminar thickness of the conductor of Ampere's bridge

References

[1] P. G. Moyssides and P. T. Pappas, J. Appl. Phys. 59, 19 (1986).
[2] A. M. Ampero, Mem, Acad. R. Sci. (paris) (1823).
[3] P. T. Pappas, Nuovo Cimento B78, 189 (1983).
[4] C. Hering, Trans. Am. Inst. Electr. Eng. 42, 311 (1923).
[5] F. F. Cleveland, Philos, Mag. 21, 416 (1936).
[6] I. A. Robertson, Philos, Mag. 36, 32 (1945).
[7] P. Graneau, J. Appl. Phys. 53, 6648 (1982).
[8] P. Graneau, Nuovo Cimento B78, 231 (1983); IEEE Trans. Magn. MAG-20, 444 (1984).
[9] A. M. Hillas, Nature 302, 271 (1983).
[10] H. Aspden, Nuovo Cimento Lett, 39, 247 (1984).
[11] G. M. Graham and D. G. Lahoz, Nature 285, 154 (1980).
[12] S. B. L. Mathur, Philos. Mag. XXXIT, 171 (1941).
[13] P. T. Pappas and P. G. Moyssides, in *Proceedings of the Third Panellenic Conference in Physics* (Hellenic Physical Society, Athens, 1983).
[14] E. F. Northrup, Phys. Rev. 24, 474 (1907).
[15] P. Graneau, Nature 295, 311 (1982).
[16] H. Aspden, J. Franklin Inst. 287, 179 (1969); Physics Unified, Sabberton, Southampton, UK (1980).
[17] J. C. Maxwell, *A Treatise on Electricity and magnetism,* Vol. 2 (Clarendon, Oxford, 1891).
[18] J. C. Ternan, J. Appl. Phys. 57, 1473 (1985).
[19] P. T. Pappas and P. G. Moyssides, Phys. Lett. **A111**, 193 (1985).
[20] J. P. Wesley, Spec. Sci. Tech. 10, *47 (1987).*