tions in Fig. 21; the second-order effect is shown by the curve next below; while the fourth curve in each instance shows the sum of the third, fourth, and fifth components. It is evident that the observed curves contain very little trace of any effects of any higher orders. The residual curves are of very small amplitude and are evidence of the fact that the incidental and random errors are small. The harmonic analysis and synthesis has been performed by methods which have been completely described elsewhere by the writer. ${ }^{12}$

The harmonic analysis of the observations gives directly the amplitude in hundredths of a fringe width and the phase as referred to the north point of the second harmonic of the curve, which is the ether-drift effect. The observed amplitude of the movement of the fringes is at once converted into the equivalent velocity of the relative motion of the earth and ether, as observed in the plane of the interferometer, by means of the relation developed in the elementary theory of the experiment:

$$
d=2 D\left(v^{2} / c^{2}\right)
$$

and

$$
v=\left(d c^{2} / 2 D\right)^{4},
$$

$d$ being the observed half-period displacement of the fringes and $D$ the length of the arm of the interferometer, that is the distance from the halfsilvered mirror by means of multiple reflections to the end mirror, No. 8, both being expressed in terms of the effective wave-length of the light used for the interferences; $v$ is the relative motion of the earth and the ether in the plane of the interferometer and $c$ is the velocity of light, both being expressed in kilometers per second. The nomograph, Fig. 20, consists of a parabolic curve which shows the relative velocity corresponding


Fig. 20. Relation of fringe displacement to velocity of etherdrift for $2 D=112,000,000 \lambda$ and $\lambda=5700 \mathrm{~A}$
${ }^{14}$ D. C. Miller, The Science of Musical Sounds, 123 (1916); J. Frank. Inst. 181, 51 (1916); 182, 285 (1916)
to a fringe displacement as observed in the interferometer used in these experiments. It is for light of wave-length $\lambda=5700 \mathrm{~A}$, and for a total light-path of $2 D=112,000,000 \lambda$. The azimuth of the ether-drift effect is the direction in which the telescope points when the half-period displacement of the fringes is a positive maximum. This azimuth, $A$, is obtained from the phase, $P$, of the second harmonic component of the observations as determined by the analyzer, from the following relation:

$$
A=(1 / 2)\left(P-90^{\circ}\right)
$$

The point thus located is the crest of the curve representing the second component, expressed in degrees, measured from the north point; the $x$-axis of the curves shown in Fig. 21 begins at the


Fig. 21. Harmonic analysis of ether-drift observations.
north point and extends for one turn of $360^{\circ}$, through the east, south and west points of azimuth back to the north point. The figure shows, in the graphs of the second components, what has already been mentioned, that, within $360^{\circ}$ of azimuth for one complete turn, there are two crests of the second component, corresponding to two azimuths $180^{\circ}$ apart, between which the interferometer is incompetent to distinguish. The dispersion of the azimuth readings is much less than $90^{\circ}$ and the necessity for continuity of azimuth indications for successive observations removes the ambiguity as to which zone of

