Are the two relativistic theories compatible?

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In a famous relativistic argument ("clock paradox") a clock U_1 is at rest in an inertial reference system S while another clock U departs from U_1 and later reverses its motion to join U_1 again. We show that the time lag of U, well understood within the special theory of relativity (STR), is more difficult to describe in reasonable physical terms in the framework of the general theory of relativity (GTR) if two clocks U_1 and U_2 in different fixed positions of S are considered. In fact, according to the GTR, the inertial forces felt in the rest frame of U modify differently the time shown by U_1 and U_2 through the action of a gravitational potential. Thus the predictions of the GTR and of the STR clash at the empirical level. The only reasonable possibility left is to give up the idea that a gravitational potential acts on the time marked by U_1 and U_2.

1. Einstein's two formulations of the clock paradox

The 1905 formulation of the clock paradox [1] had a possible implication that surely Einstein did not like. The differential time lag is an absolute effect, as all observers agree about the time marked e.g. by the clock which has moved with variable velocity, when the two clocks reunite. However, they disagree about the numerical value of this variable velocity at any position of the clock in space. In relativity all inertial observers (forming an infinite set) are *completely* equivalent, so that, in a sense, one can say that the clock velocity can assume simultaneously all conceivable values. Yet a quantity having infinitely many values is totally undefined. In this way the presumed cause of the time lag (velocity) seems to disappear into nothingness. This is not physically reasonable, as obviously the cause of a real physical phenomenon must also be concrete, in spite of the evasive description deduced from the theory. Therefore causality implies that velocity itself should be well defined, that is, relative to a physically active reference background, which defines at the same time a privileged reference system.

It is no surprise, then, that to escape from such conclusions the original formulation of the clock paradox argument was completed with a second formulation based on the general theory of relativity [2], whose essential points we will now review.

Let S be an inertial reference system. Further, let U_1 and U be two exactly similar clocks working at the same rate when at rest near to one another. If one of the clocks - let us say U - is in a state of uniform translatory motion relative to S, then, according to the STR it works more slowly than U_1, which is at rest in S. At this point Einstein adds an interesting remark: "This result seems odd in itself. It gives rise to serious doubts when one imagines the following thought experiment." In Einstein's thought experiment O is the origin of S, and Q a different point of the positive x-axis. The two clocks are initially at rest at O, so that they work at the same rate and their readings are the same. Next, a constant velocity in the direction $+x$ is imparted only to U, so that it moves towards Q. At Q the velocity is reversed, so that U returns towards O. When it arrives at O its motion is stopped, so that it is again at rest near U_1. Since U works more slowly than U_1 during its motion, U must be behind U_1 on its return.

Now comes the problem. According to the principle of relativity the whole process must surely take place in exactly the same way if it is considered in a reference system S_a sharing the movement of U. Relatively to S_a, it is U_1 that executes the to and fro movement while U remains at rest throughout. From this it would seem to follow that, at the end of the process, U_1 must be behind U, a conclusion incompatible with the previous result.

But, Einstein says, the STR is not applicable to the second case, as it deals only with inertial reference systems, while S_a is at times accelerated. Only the GTR deals with accelerated systems. From the point of view of the GTR, one can use the coordinate system S_a just as well as S. But in describing the whole process, S and S_a are not equivalent as the following comparison of the relative motions shows.

S* *Reference System

1. The clock U is accelerated by an external force in the direction $+x$ until it reaches the velocity v. U_1 remains at rest, now as in the following four steps.

2. U moves with constant velocity v to the point Q on the $+x$-axis.

3. U is accelerated by an external force in the direction $-x$ until it reaches the velocity v in the direction $-x$.

4. U moves with constant velocity v in the direction $-x$ back to the neighbourhood of U_1.

5. U is brought to rest by an external force very near to U_1.

S_a* *Reference System

1. A gravitational field, oriented along $-x$, appears, in which the clock U_1 falls with an accelerated motion until it reaches the velocity v. When U_1 has reached the

velocity v the gravitational field vanishes. An external force applied to U prevents U from being moved by the gravitational field.

2. U_1 moves with constant velocity v to a point Q' on the $-x$-axis. U remains at rest.

3. A homogeneous gravitational field in the direction $+x$ appears, under the influence of which U_1 is accelerated in the direction $+x$ until it reaches the velocity v, whereupon the gravitational field vanishes. U is kept at rest by an external force.

4. U_1 moves with constant velocity v in the direction $+x$ into the neighbourhood of U. U remains at rest.

5. A gravitational field in the direction $-x$ appears, which brings U_1 to rest. The gravitational field then vanishes. U is kept at rest by an external force.

The second description is based on the principle of equivalence between fictitious and gravitational forces. According to both descriptions, at the end of the process the clock U is retarded by the same amount with respect to U_1. With reference to S_a this is explained by noticing that during the stages 2 and 4, the clock U_1, moving with velocity v, works more slowly than U, which is at rest. But this retardation is overcome by the faster working of U_1 during stage 3. For, according to the GTR, the higher is the gravitational potential in the region where a clock is placed, the faster the clock works. During stage 3 U_1 is indeed in a region of higher gravitational potential than U. A calculation made with instantaneous acceleration shows that the consequent advancement amounts to exactly twice as much as the retardation during stages 2 and 4 [2], so that the final prediction coincides with that obtained in S. Arrived at this conclusion Einstein states: "This completely clears up the paradox."

The prediction of the GTR that a clock works faster the larger the gravitational potential ϕ in the region at which it is placed is confirmed by the experiments performed in the gravitational field of the Earth, so that at first sight the 1918 reasoning could seem to be a consequence of empirical facts. The mathematical treatment of the clock paradox situation given by the GTR leads to the right result by describing the retardation of U as a consequence of the action of ϕ on U_1 and U_2 [3]. Yet the theory shows its weakness in other ways [4] as the arguments developed in the final part of the present paper clearly show.

In the next section the STR description of the clock paradox is reviewed for the case of two clocks (U_1 and U_2) constantly at rest on the x axis of the inertial system S, while a third clock, U, performs a to and fro motion on the same axis. We adopt a simplified version of the U motion in which there is acceleration only at the turning around point. The differential retardation due to the to and fro motion is then obtained, e.g. by subtracting the retardation at the first U_1-U meeting from the total retardation at the second meeting. The description is straightforward and of course the stationary clocks U_1 and U_2 are predicted to maintain the synchronization they had before U was moved. In the third section the clock paradox for three clocks is discussed anew from the point of view of U considered at rest in the non inertial reference system S_a. The GTR seems at first to explain the observations, but this time the price to pay is to accept that the synchronization of U_1 and U_2 be differently modified by the gravitational potential ϕ. The last section will comment this result and conclude that it clashes with the predictions of the STR at the empirical level. In this way it becomes necessary to give up the idea that associated with the fictitious forces felt in the rest frame of U there is a gravitational potential acting on the time marked by U_1 and U_2.

2. Invariant retardations from the point of view of the inertial system

Three clocks are given, U_1, U_2 and U, which mark time in the same way if at rest with respect to one another. The first two are constantly at rest on the x axis of the inertial reference system S, at points with respective coordinates x_1 and x_2 ($x_2 > x_1 > 0$). The third clock, U, moves on the same axis, initially in the $+x$ direction with constant velocity v (see Fig. 1). It was set at the time $t' = 0$ when it passed by the origin O of the coordinate system S, whose observers also adopted the time $t = 0$ when U passed by O. Of course U can be considered at rest in the origin of an inertial reference system S' before the accelerated motion starts. The Lorentz transformations from S to S' can then be written

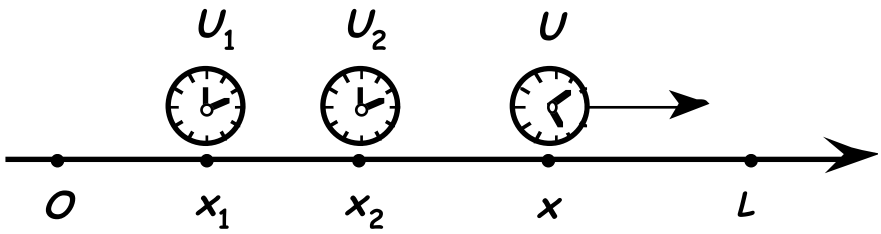

Figure 1. Two clocks, U_1 and U_2, are constantly at rest on the x axis. A third clock, U, moves on the same axis with constant velocity v. Arrived at the point with coordinate L the clock U reverses its motion moving thereafter with constant velocity $-v$.

$$\begin{cases} x' = \dfrac{x - vt}{R} \\ y' = y \qquad ; \qquad z' = z \\ t' = \dfrac{t - xv/c^2}{R} \end{cases} \tag{1}$$

where

$$R = \sqrt{1 - v^2/c^2} \tag{2}$$

The equations of motion of U seen from S are $x = vt$ and $y = z = 0$, so that Eqs. (1) give

$$x' = y' = z' = 0 \qquad \text{and} \qquad t' = Rt \tag{3}$$

meaning that U remains constantly in the origin of S' and marks a time smaller by a factor R with respect to the S clocks. Thus the STR predicts that U_1 and U, meeting for the first time at x_1, mark respectively the (proper) times

$$\tau_1 = \frac{x_1}{v} \qquad ; \qquad \tau'_1 = \frac{x_1}{v} R \tag{4}$$

Therefore, passing from $x = 0$ to $x = x_1$, U accumulates the retardation

$$T(0, x_1) \equiv \tau'_1 - \tau_1 = -\frac{x_1}{v}(1 - R) \tag{5}$$

with respect to U_1. Similarly, the STR predicts that U_2 and U, meeting for the first time at x_2, mark respectively the (proper) times

$$\tau_2 = \frac{x_2}{v} \qquad ; \qquad \tau'_2 = \frac{x_2}{v} R \tag{6}$$

Therefore, passing from $x = 0$ to $x = x_2$, U accumulates the retardation

$$T(0, x_2) \equiv \tau'_2 - \tau_2 = -\frac{x_2}{v}(1 - R) \tag{7}$$

with respect to U_2. The times (4) and (6), as well as the retardations (5) and (7), are invariant for all observers. That it must be so is obvious for general physical grounds; for the reading on any clock of the time of an event coincident with it must be the same for all observers. The retardations (5) and (7), stored up before the first meetings of U with U_1 and U_2, are not essential for a "clock paradox" reasoning in which the crucial term is the retardation accumulated between the first and the second meeting. Therefore $T(0, x_1)$ and $T(0, x_2)$, considered for clarity only, will later be subtracted away in order to get the physically relevant retardations.

When U, continuing in its motion, reaches the point with coordinate L ($L > x_2 > x_1$), U_1 and U_2 mark the time $t = L/v$ while U marks the proper time

$$\tau'_L = \frac{L}{v} R \tag{8}$$

Clearly the proper times spent by U in covering the distances $L - x_1$ and $L - x_2$ are

$$\tau'_L - \tau'_1 = \frac{L - x_1}{v} R \qquad ; \qquad \tau'_L - \tau'_2 = \frac{L - x_2}{v} R \tag{9}$$

respectively. Also the L dependent times (8 - 9) are invariant, for the reading on U of the time of its superposition with the point of S with coordinate L has to be the same for all observers.

Having reached the point with coordinate $x = L$, U reverses instantaneously its velocity from v to $-v$. We suppose that the times required to accelerate or decelerate U are so small that they can be neglected without appreciable error. This can always be realized, even for moderate acceleration, by supposing L to be very large as was shown by Dingle [5], or it may be justified for rapid accelerations [6]. Therefore the proper times $\Delta\tau'_1$ and $\Delta\tau'_2$ spent by U on the distances $2(L - x_1)$ and $2(L - x_2)$ covered when U meets for the second time U_1 and U_2, respectively, are doubled with respect to (9):

$$\Delta\tau'_1 = 2\frac{L - x_1}{v}R \qquad ; \qquad \Delta\tau'_2 = 2\frac{L - x_2}{v}R \tag{10}$$

Meanwhile, U_1 and U_2 have stored up the proper times

$$\Delta\tau_1 = 2\frac{L - x_1}{v} \quad ; \quad \Delta\tau_2 = 2\frac{L - x_2}{v} \tag{11}$$

The retardations are the differences between the proper times (10) and (11):

$$\begin{cases} T_1 \equiv \Delta\tau'_1 - \Delta\tau_1 = -2\dfrac{L - x_1}{v}(1 - R) \\ \\ T_2 \equiv \Delta\tau'_2 - \Delta\tau_2 = -2\dfrac{L - x_2}{v}(1 - R) \end{cases} \tag{12}$$

The differential retardations (12) of the U clock are those for which the "clock paradox" proper is usually considered. They both refer to a time interval during which a clock separates from, and later reunites with, a stationary clock.

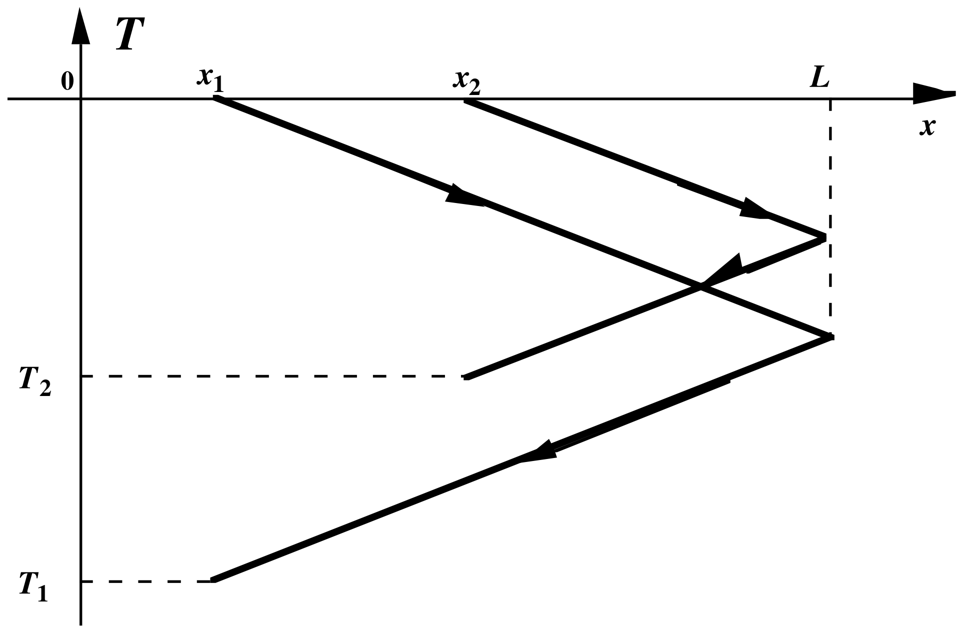

Figure 2. Invariant differential retardations of the travelling clock U as functions of its position relative to the rest system of the stationary clocks U_1 and U_2, as predicted by the STR.

As stated above the clocks U_1 and U_2 are constantly at rest at the points x_1 and x_2 on the x axis of the inertial reference system S. Obviously the rates of U_1 and U_2 are not affected by the motion of U, as the STR is based on the implicit idea that motions of clocks, whether uniform or accelerated, cannot modify a preestablished synchronization of other clocks. Therefore, U_1 and U_2 are not influenced by U when it moves from the origin to the point with coordinate L and back. Of course, the velocity retardations (12) are different if $x_1 \neq x_2$, as we assumed. This difference arises from to and fro paths of unequal length $[2(L-x_1)$ vs. $2(L-x_2)]$ and duration $[2(L-x_1)/v$ vs. $2(L-x_2)/v]$ covered by the travelling clock U to pass near U_1 and U_2 twice. In particular the latter clocks maintain the same synchronization before, during and after the trip made by U. A different assumption would lead to disagreement with the STR. The retardations (12), just as the clock readings from which they are deduced, are invariant, that is they are the same for all observers, given that they correspond to objective events. In fact when two clocks overlap any observer, however he/she may be moving, must necessarily see the same readings on the displays.

3. Invariant retardations from the point of view of the accelerated system

Let us discuss the clock paradox in a noninertial reference system from the point of view of the general theory of relativity (GTR). Of our three clocks, U_1, U_2 and U, the third one is constantly at rest in the generally noninertial reference system S_a, while the first two perform a to and fro movement. The travellers are supposed to set out from U at uniform speed and, after a while, to reverse their motion and go back along the same path keeping the same speed.

As is well known, the GTR reduces to the STR when the acceleration of the reference system goes to zero. Therefore the GTR describes the effects of the paths covered by U_1 and U_2 with constant velocity exactly as the STR would, and obtains naturally conclusions *opposite* to those given by (12), deduced in S. Thus the constant velocity paths give rise to invariant differential retardations of the moving clocks U_1 and U_2 with respect to the stationary clock U respectively given by

$$T'_{1,vel.} = T_1 \qquad ; \qquad T'_{2,vel.} = T_2 \tag{13}$$

where the primes indicate that a result is obtained according to observers at rest in S_a. Notice that the quantities (13) [retardations of U_1 and U_2 with respect to U] are indeed physically opposite to those satisfying (12) [retardations of U with respect to U_1 and U_2]. The quantities in the right hand sides of (13) are relative to S. The mixed notation is made possible by the invariance of the retardations and is adopted for easier comparison with the previous section. Once more, we stress that the different values of the retardations (13) do not imply any new synchronization of the moving clocks, the difference being due to the unequal lengths of the two way trips executed by U_1 and U_2 to pass near U twice. All this is consistent with the STR, of course.

According to the GTR one must also consider the equivalence between inertial and gravitational forces and add the effects of the potential $\phi(x')$ of the constant (in space) gravitational field perceived in the rest frame of U at the time of acceleration. It was shown by Einstein [2] and Møller [3] that such effects show up in U_1 and U_2 and have opposite sign and double value of the velocity effects given by (13). That is

$$T'_{1,\phi} = -2\,T_1 \qquad ; \qquad T'_{2,\phi} = -2\,T_2 \tag{14}$$

This result was confirmed by Iorio [7], who considered a finite time of acceleration. If the GTR describes reality, the above positive retardations (thus, *anticipations*) are *concrete sudden modifications* of the time shown by the clocks U_1 and U_2, simultaneous (in S_a) with the appearence of the inertial forces acting on U. Of course the gravitational anticipations (14) consist of an istantaneous addition in the displays of U_1 and U_2 of the times $T'_{1,\phi}$ and $T'_{2,\phi}$, respectively.

The total time variations of U_1 and U_2 accumulated between the first and the second meeting with U and calculated in S_a are respectively given by

$$T'_1 = T'_{1,vel.} + T'_{1,\phi} \qquad ; \qquad T'_2 = T'_{2,vel.} + T'_{2,\phi} \tag{15}$$

In Fig. 3 the broken lines $O'A_1B_1C_1$ and $O'A_2B_2C_2$ represent the evolution of the retardations of the travelling clocks U_1 and U_2 with respect to the clock U, which is considered at rest. The slanting segments $O'A_1$, $O'A_2$, B_1C_1, B_2C_2 represent the retardations due to the (constant) velocity of U_1 and U_2. The vertical segments A_1B_1 and A_2B_2, whose different lengths provide the main argument of this paper, represent the anticipations due to the gravitational potential of the fictitious forces.

From (13) – (15) one gets the expected results

$$T'_1 = -T_1 \qquad ; \qquad T'_2 = -T_2 \tag{16}$$

as necessary, given the invariant nature of the retardations.

Notice that all the retardations and anticipations introduced in the present section are invariant. In fact, one can see from (13), (14) and (16) that they are proportional with simple numerical factors to T_1 and T_2, proven invariant in the previous section. The invariance of the gravitationally produced modifications of the times marked by U_1 and U_2 can also be shown in a direct way as follows. As we saw, from the point of view of the inertial reference system S, when the moving clock U reaches the point $x = L$ at time $t = L/v$ it marks the (proper) time $t' = RL/v$. From the point of view of the system S_a the birth of the gravitational potential ϕ is instantaneous as soon as the moving point of S with coordinate $x = L$ superimposes on U. Therefore the clocks U_1 and U_2 are just about to mark the times

$$t'_1 = t'_2 = RL/v$$

when the potential ϕ produces jumps forward [given by (14)] of the times marked by U_1 and U_2. These jumps imply the immediate consecutivity in time

$$\text{between} \qquad t'_1 = RL/v - \varepsilon \qquad \text{and} \qquad t'_1 = RL/v + T'_{1,\phi}$$

and

$$\text{between} \qquad t_2' = RL/v - \varepsilon \qquad \text{and} \qquad t_2' = RL/v + T_{2,\phi}'$$

with $\varepsilon > 0$ as small as one wishes. This proves the invariance of the gravitationally produced modifications of the times marked by U_1 and U_2. In fact observers with different velocities, e.g. passing near U_1, must see the same development on the display of U_1, namely the time $t_1' = RL/v - \varepsilon$ immediately followed by a jump by $T_{1,\phi}'$, as this phenomenon is objectively real. Such modifications of U_1 and U_2 are thus seen to be the same from all reference systems, in particular from S, the rest system of the two clocks.

In conclusion, it can be said that the GTR leads to the correct prediction of the differential retardation of two clocks that separate and then join again. Its treatment however gives rise to a serious difficulty, as we will see next.

4. Critical remarks

It is important to understand whether the formulation of the clock paradox given by the GTR can describe reality. In other words whether all consequences of the theory, besides those usually discussed, are compatible with empirical evidence.

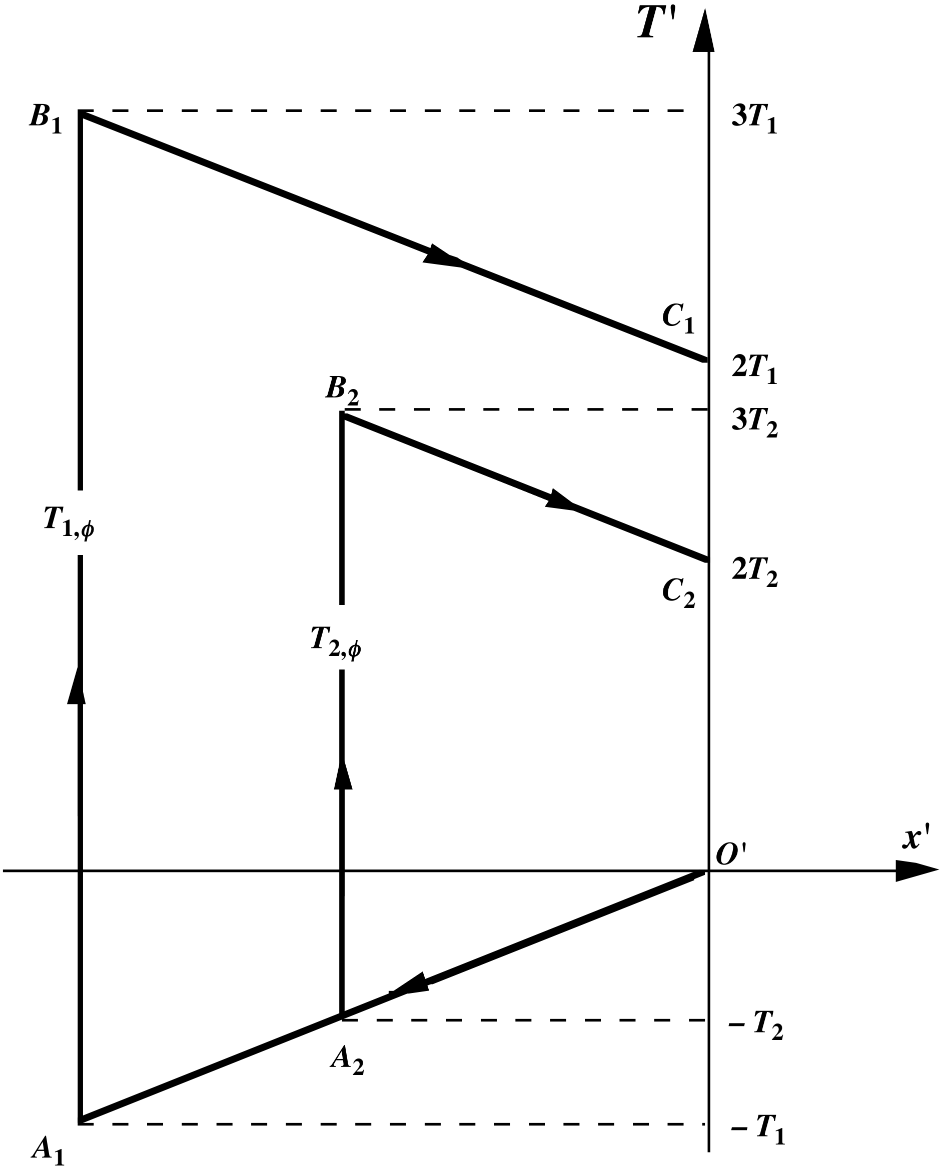

Figure 3. The broken lines $O'A_1B_1C_1$ and $O'A_2B_2C_2$ give the evolution of the invariant differential retardations of the travelling clocks U_1 and U_2 with respect to the clock U (considered at rest) as predicted by the GTR. The slanting segments represent the retardations due to velocity, while the vertical segments A_1B_1 and A_2B_2 represent the anticipations due to the gravitational potential of the fictitious forces.

As stated above the clocks U_1 and U_2 are constantly at rest at the points x_1 and x_2 on the x axis of the inertial reference system S. We can be sure, that the rates of U_1 and U_2 are not affected by the rectilinear uniform motion of U, as the STR is based on the implicit idea that motions of clocks do not modify a preestablished synchronization of other clocks. Therefore, U_1 and U_2 are not influenced by U when it moves from the origin to the point with coordinate L and back. As we saw the different values of the velocity retardations (13) are easily explained by their arising from to and fro paths of different length and duration of the travelling clocks.

The slanting segments of Fig. 3 ($O'A_1$, $O'A_2$, B_1C_1 and B_2C_2) are symmetrical of those of Fig. 2. Notice in fact that if the vertical segments A_1B_1 and A_2B_2 are shortened till they reach zero length, keeeping A_1 and A_2 fixed, the retardations of Fig. 3 become somehow the specular images of the retardations of Fig. 2. This is no surprise as the uniform motions seen from S and S_a are perfectly symmetrical.

In the theory of general relativity the presence and the size of the gravitational effect is truly essential. Without it one would fall back to the situation in which the whole process takes place exactly in the symmetrical way if considered in the reference system S_a sharing the movement of U. From this it would follow, at the second meeting with U, that U_1 and U_2 would be retarded with respect to U, an unacceptable conclusion as it clashes with the predictions obtained in S. Besides, we know for sure that actually U will be retarded [8].

Nevertheless the anticipations (14), due to the potential, give rise to a serious difficulty. Their different values, due to the diversity of $\phi(x_1)$ and $\phi(x_2)$ imply a desynchronization of U_1 and U_2 similarly to the case of clocks in the gravitational field of the Earth, which change observably their pace every time the altitude is modified. Naturally, if the hands of a clock undergo a sudden change of position when the clock is just overlapping the point x' of the S' coordinate system, all the observers, independently of their state of motion, must agree on the reality of the jump. There is no way to assume that the jump is real only for an observer. Therefore, if the clocks undergoing sudden jumps of the previous type are two, and if the jumps are different, the resulting desynchronization must be real for all observers, in particular for those at rest with the clocks.

In the STR U_1 and U_2 are synchronized in their rest system S in such a way as to measure from x_1 to x_2 a velocity of light equal to c. This is certainly true before U accelerated, but it cannot be so anymore after the anticipations $T'_{1,\phi}$ and $T'_{2,\phi}$ have set in. For, it must be stressed once more that if the times shown in the displays of U_1 and U_2 are modified in S_a, the modification must be an objectively real phenomenon so that all the observers have to agree about it. A different assumption would violate the firmly established invariance of the retardations and would make eq. (16) impossible. We can conclude that the clock readings are modified in all reference systems, in particular in S. It follows that if before the acceleration of U a flash of light left x_1 at time t_1^* (marked by U_1) and arrived in x_2 at time t_2^* (marked by U_2) such that

$$\frac{t_2^* - t_1^*}{x_2 - x_1} = \frac{1}{c} \tag{19}$$

after the acceleration of U one will instead detect a fictitious velocity of light $\tilde{c}$ given by

$$\frac{1}{\tilde{c}} = \frac{\left[t_2^* + T'_{2,\phi}(0,0)\right] - \left[t_1^* + T'_{1,\phi}(0,0)\right]}{x_2 - x_1} \tag{20}$$

From (20), using (14), one gets

$$\frac{1}{\tilde{c}} = \frac{1}{c} - \frac{4}{c}\,\frac{1-\sqrt{1-v^2/c^2}}{v/c} \tag{21}$$

The correction is not negligible. For example, for small values of v/c one has

$$\tilde{c} \cong c\left(1 + 2\frac{v}{c}\right) \tag{22}$$

According to the accelerated observer, the change in synchronization reflected in the difference between (19) and (21) has to be simultaneous with the fictitious forces felt in the system S_a. As we saw, it is possible to predict the exact time at which such resynchronization becomes effective in the rest system of U_1 and U_2. It is however enough to point out that the GTR obviously predicts that in all inertial systems such time will be *before* the second meeting of U with U_1 and U_2. Therefore an ideal experiment devised to discriminate (21) from (19) should be made while U keeps moving with uniform motion towards $-\infty$ after its second meeting with U_1 and U_2. It is remarkable that according to the STR the invariant resynchronization predicted by the GTR does not take place and the velocity of light given by (19) applies both before and after the acceleration of U. In this respect the two theories are incompatible and cannot both be right.

Although it is probably possible to check (21) experimentally, this may not be necessary, as it is hard to believe that something arising from the acceleration of a clock can really modify the time marked by other clocks situated far away from it. It is easier to believe that the potential of the fictitious forces $\phi(x')$ does not have any effect on the time marked by clocks. After all, according to the equivalence principle, $\phi(x')$ does not describe a gravitostatic field like that of the Earth, but arises from accelerated motions. According to Einstein $\phi(x')$ acts like an ordinary field, but on this point he was probably not right, in spite of the possible general correctness of the equivalence principle. In our view the "equivalence" has to be seen as a kinship and not as an identity. Analogous is the case of the magnetic field, dynamical manifestation of the electric field, but with different interaction properties

Therefore it seems likely that the GTR describes only a game of appearances and not a real physical process. Anybody respecting the notion of objective reality, at least at the macroscopic level, can only agree with Builder [6]: *"The principle of equivalence is completely irrelevant to analysis and discussion of the relative retardation of clocks unless there is a real gravitational field to be taken into account and, except in such a case, the general theory of relativity can add nothing of physical significance to an analysis correctly made using the restricted theory."*

The elimination of the gravitational effects leads to a resolution of the clock paradox which may not only be logically possible, but also intellectually satisfactory [4]. In discussing the differential retardation of clocks Einstein appears to have been closer to the truth in 1905 than in 1918.

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