This book reviews the results obtained in recent years by the author in relativistic physics. The recently increased conviction about the conventional definition of relativistic simultaneity has opened the doors to new ideas, in spite of the fact that research has shown that simultaneity in the physical reality exists and is not at all conventional. If the coefficient of the space variable $x$ in the Lorentz, or other, transformation of time (we call it $e_1$) had a conventional nature it should be possible to modify it without touching the empirical predictions of the theory. Given that Einstein’s principle of relativity leads necessarily to the Lorentz transformations, and thus also to a fixed value of $e_1$, such a modification would imply a reformulation of the relativistic idea itself. With respect to the idealized initial picture, the concrete development of research has produced some exciting novelties. Several phenomena, in particular those taking place on accelerating systems (Sagnac effect, and all that) converge in a strong indication of the value $e_1=0$. This implies absolute simultaneity and a new type of space and time transformations, which we call "inertial". We give six proofs of absolute simultaneity, which are essentially independent of one another. In order to make their identification easy, the six chapters in which these proofs are given have the equality $e_1=0$ already in the title. The cosmological consequences of the new structure of space and time go against the big bang model. After our results relativism, although weakened, is not dead, but survives in milder forms.
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Chapter 1

Einstein positivist/realist

Einstein’s 1905 philosophical standpoint was as follows: ether does not exist, therefore it does not make any sense to consider motion with respect to nothing. Motion has to be described with respect to concrete systems only and a complete symmetry exists between the conclusions of different inertial observers. Considering a clock in motion relative to the inertial observers \( O_1; O_2; \ldots O_n \) with respective velocities \( v_1; v_2; \ldots v_n \), its rate should appear slowed by the respective factors \( R(v_1); R(v_2); \ldots R(v_n) \), given by a unique function of relative velocity, in agreement with the relativity principle. A legitimate question remains: “What really happens to the clock, which is it its true rate?” The relativistic answer is that this question does not make sense, and that the conclusions of all the different observers are equally valid. The argument can be generalized to any other physical quantity.

The impact of Mach’s positivism was transmitted by the young Einstein. But around 1920 Einstein turned away from positivism because he realized with a shock some of its consequences; consequences which the next generations of brilliant physicists (Bohr, Pauli, Heisenberg) not only discovered but enthusiastically embraced: they became subjectivists. But Einstein’s withdrawal came too late. “Physics had become a stronghold of subjectivist philosophy, and it has remained so ever since.” [Popper]. Popper witnessed Einstein’s radical change of opinion about Mach’s philosophy: “Einstein himself was for years a dogmatic positivist ... He later rejected this interpretation: he told me in 1950 that he regretted no mistake he ever made as much as this mistake.”

A conceptual clash around the idea of physical reality accompanied and followed the birth of the Copenhagen-Göttingen quantum theory. The realists, headed by Einstein, included Planck, Ehrenfest, Schrödinger and de Broglie. Winners were however the antirealists (Bohr, Born, Heisenberg, Pauli, Dirac) not because they could prove the realists’ ideas false, but because they were united in developing a theory coherent with their ideal choices and able to explain a remarkable number of phenomena. Hans Reichenbach wrote: “You say that while you are in your office your house stands unchanged in its place. How do you know? [...] The trouble is that unless you can find a better answer to that question than is supplied by common sense, you will not be able to solve the problem of whether light and matter consist of particles or waves.” The idealism of quantum theory was too much for Einstein, as his relativism was surely a milder form of rejection of the objective reality than provided by the Copenhagen doctrine. Einstein never accepted the final formulation of quantum mechanics.
1. Einstein positivist/realist

Mach’s epistemology had a strong impact at the end of the XIXth and the beginning of the XXth century. One can say that the theory of relativity was formulated by trying to satisfy at least part of the epistemological demands of the Viennese philosopher, which are so described by Kostro: “Notions such as ‘force,’ ‘matter,’ ‘atom,’ ‘absolute space’ are our subjective inventions, not something experimentally tangible. They should therefore be eliminated from physics. After this, one would be left only with “sense impressions,” which Mach preferred to call “elements.” What we call the world is nothing but the system of such “elements”.” [1-1] From the theory of special relativity (TSR) Einstein deduced that every clock in motion slows the pace of its time. His 1905 standpoint was the following: ether does not exist, therefore it does not make any sense to consider motion with respect to nothing. Motion has to be described with respect to concrete systems only. The slowing down of clocks is always relative to observers who see them in motion, and a complete physical and philosophical symmetry exists between the conclusions of different inertial observers. Considering a clock in motion relative to the inertial observers $O_1; O_2; ... O_n$ with respective velocities $v_1; v_2; ... v_n$, its rate should appear slowed by the the respective factors $R(v_1); R(v_2); ... R(v_n)$, given by a unique function of relative velocity, in agreement with the relativity principle. A legitimate question seems to remain: “What really happens to the clock, which is it its true rate?” The relativistic answer is that this question does not make sense, and that the conclusions of all the different observers are equally valid. In this way the philosophy of relativism and subjectivism becomes dominant in physics for the observations of the inertial observers. Of course the argument can be generalized by going from the time marked by clocks to any other physical quantity: we will see it done by the English physicist J. Jeans in the third section.

Einstein’s relativism clearly originates in positivism and, in a way, it is surprising that it was never disavowed by the founder of relativity in spite of his sharp break with Mach. “The fact that Mach condemned the theory of relativity was a very unpleasant experience for Einstein. He stopped praising Mach’s achievements and started criticizing him and his epistemological views” [1-2] To show this point, Kostro quotes Einstein’s answer to a question asked by Emil Meyerson during a reception on April 6, 1922 in Paris, organised by the French Philosophical Society in honour of Albert Einstein. : “There does not appear to be a great relation from the logical point of view between the theory of relativity and Mach’s theory. [...] Mach’s system studies the existing relations between data of experience; for Mach science is the totality of these relations. That point of view is wrong, and, in fact, what Mach has done is to make a catalogue, not a system. To the extent that Mach was a good mechanician he was a deplorable philosopher.” “[1-3]

The critical point of view of Mach’s philosophy was kept till the end, as one can see from the 1948 Scientific Autobiography [1-4] where Einstein expressed an appraisal of the Machian philosophy very similar to the previous one. Ten years before, in a letter to M. Solovine Einstein had stated: “In these days the subjective and positivist viewpoint dominates in a most excessive manner. The need for conceiving nature as an objective reality is declared to be an obsolete prejudice, and thus a virtue is made of the necessity of quantum theory. Men are just as subject to suggestion as horses, and each epoch is dominated by a fashion, and the majority do not even see the tyrant who dominates them.” [1-5] Einstein’s criticism of positivism is surely deep and interesting, nevertheless it is difficult to avoid the impression that he underestimated its impact on his own scientific creations.

It is worth recalling that important epistemologists shared Einstein’s critical evaluation of positivism. E.g., Karl Popper wrote: “Positivists [...] are constantly trying to prove that
metaphysics by its very nature is nothing but nonsensical twaddle - sophistry and illusion - as Hume says, which we should commit to the flames. [...] There is no doubt that what the positivists really want to achieve is not so much a successful demarcation as the final overthrow and the annihilation of metaphysics.” [1-6] and: “Positivists, in their anxiety to annihilate metaphysics, annihilate natural science with it. For scientific laws ... cannot be logically reduced to elementary statements of experience. If consistently applied, Wittgenstein’s criterion of meaningfulness rejects as meaningless those natural laws the search for which, as Einstein says, is the supreme task of the physicist” “[1-7] Popper ascribed the spreading of positivism in physics to the influence of the young Einstein. A statement that seems to me to go to the core of the problem is the following: “The philosophical impact of Mach’s positivism was largely transmitted by the young Einstein. But Einstein turned away from Machian positivism, partly because he realized with a shock some of its consequences; consequences which the next generation of brilliant physicists, among them Bohr, Pauli and Heisenberg, not only discovered but enthusiastically embraced: they became subjectivists. But Einstein’s withdrawal came too late. Physics had become a stronghold of subjectivist philosophy, and it has remained so ever since.” [1-8]. In fact Popper could witness Einstein’s radical change of opinion about Mach’s philosophy: “It is an interesting fact that Einstein himself was for years a dogmatic positivist and operationalist. He later rejected this interpretation: he told me in 1950 that he regretted no mistake he ever made as much as this mistake.” [1-9]

It is fair to add that Einstein described himself as oscillating between different philosophies. He devoted many papers to epistemology. Other famous physicists published articles and books on the same argument (Planck, Schrödinger, Bohr, Heisenberg), but nobody with the richness and the critical skill of Einstein. About the relationship between physics and philosophy he wrote: “The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is - insofar as it is thinkable at all - primitive and muddled. ... He [the scientist] accepts gratefully the epistemological conceptual analysis; but the external conditions, which are set for him by the facts of experience, do not permit him to let himself be too much restricted in the construction of his conceptual world by the adherence to an epistemological system. He therefore must appear to the systematic epistemologist as a type of unscrupulous opportunist: he appears as realist insofar as he seeks to describe a world independent of the acts of perception; as idealist insofar as he looks upon the concepts and theories as the free inventions of the human spirit (not logically derivable from what is empirically given); as positivist insofar as he considers his concepts and theories justified only to the extent to which they furnish a logical representation of relations among sensory experiences. He may even appear as Platonist or Pythagorean insofar as he considers the viewpoint of logical simplicity as an indispensable and effective tool of his research”. [1-10]

The philosophical standpoint of physicists is rarely the eclectic one here described. Different scientists embraced different philosophies, but almost always very well defined for every single author. The previous description should rather be understood in the autobiographical sense, as Einstein in different moments of his scientific activity indeed followed different philosophical ideas. I insist to say that he conformed to positivism when the two relativistic theories were formulated and defended their interpretation based on relativism during his whole lifetime. He behaved as a realist, however, in other famous papers of 1905 (on Brownian motion and on the light quanta) and in his long battle against the Copenhagen formulation of quantum mechanics.

The most essential conflict that accompanied and followed the birth of the Copenhagen-Göttingen theory was a philosophical clash around the idea of physical reality. The realists,
headed by Einstein, included Planck, Ehrenfest, Schrödinger and de Broglie. Winners were however the antirealists (Bohr, Born, Heisenberg, Pauli, Dirac) not because they could prove the realists’ ideas false, but because they were united in developing a theory coherent with their philosophical choices and able to explain a remarkable number of phenomena. A philosopher who always defended the orthodox position is our expert of relativistic synchronization, Hans Reichenbach, who wrote: “You say that while you are in your office your house stands unchanged in its place. How do you know? [...] The trouble is that unless you can find a better answer to that question than is supplied by the arguments of common sense, you will not be able to solve the problem of whether light and matter consist of particles or waves.” [1-11] The strong idealistic taste of quantum theory was confirmed by many writers, e.g. by Karl Popper: "... the Copenhagen interpretation of quantum mechanics, is about universally accepted. In brief, it says that objective reality has evaporated, an that quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles. [Popper's italics] [1-12] The degree of idealism of quantum theory was too much to bear for Einstein, as his relativism was surely a milder form of rejection of the objective reality than provided by the Copenhagen doctrine. Anyway, Einstein never accepted the final formulation of quantum mechanics, which he considered at least as incomplete as classical thermodynamics (in so far as not based on atomism). The Copenhagen physicists had the feeling to have faithfully pursued the path, which Einstein had shown adopting positivism and relativism, while he himself stopped at a certain point. 

Einstein published comments of sharply realistic mould in the context of quantum theory. For example: “There is such a thing as the ‘real state’ of a physical system, which exists objectively, independently of any observation or measurement, and which can be described, in principle, with the means of description afforded by physics.” A few lines below he added: “All men, the quantum theoreticians included, actually stick steadfastly to this thesis on reality, as long as they do not discuss the foundations of quantum theory.” And right afterwards he wrote: “I am not ashamed to make the ‘real state of a system’ the central concept of my approach.” [1-13] Einstein insisted that the physicist should try to form an image of the studied process, almost a hypothetical picture that can acquire validity only after many controls and which must be taken as the basis of the theoretical constructions. In a letter to Born of 1947 he wrote: “Therefore I cannot seriously believe in it [in quantum mechanics], because the theory is incompatible with the idea that physics should describe a reality in time and space without spookish actions at a distance.” [1-14]

Einstein fought another fundamental battle in the defense of causality: “Even the great initial success of the quantum theory does not make me believe in the fundamental dice-game, although I am well aware that our younger colleagues interpret this as a consequence of senility. No doubt the day will come when we will see whose instinctive attitude was the correct one.” [1-15] This statement of 1944 joins coherently what Einstein had written twenty years before in a letter to Born: “Bohr’s opinion about radiation is of great interest. But I should not want to be forced into abandoning strict causality without defending it more strongly than I have so far. I find the idea quite intolerable that an electron exposed to radiation should choose [of its own free will], not only its moment to jump off, but also its direction. In that case, I would rather be a cobbler, or even an employee in a gaming-house, than a physicist.” [1-16]

Thus Einstein defended realism, causality and description in space and time, against those physicists of Copenhagen and Göttingen who believed to have only continued on the path he had indicated with relativity. From this comes out all the richness, but also the complexity, of the Einsteinian conceptions. Reckoning with these ideas means entering in the eye of the epistemological-scientific storm of the XXth century. [1-17]
[1-1] [LK, p. 21]
[1-2] [LK, p. 104]
[1-4] See the book [AB]
[1-5] Letter to M. Solovine of 1938
[1-6] [SD, pp. 35-36]
[1-7] [SD, p. 36]
[1-8] [UQ, pp. 152-153]
[1-9] [UQ, pp. 96-97]
[1-10] [AB, pp. 683-684]
[1-11] [ST, p. 96]
[1-12] [MB, p. 164]
[1-13] [DB, p. 7]
[1-14] [EB, p. 116]
[1-15] [EB, p. 149]
[1-16] [EB, p. 82]
[1-17] See the book [QR]
Chapter 2

Relativistic paradoxes

The great successes of the relativistic theories are very well known. Nevertheless, it would not be correct to conclude that every comparison of the theoretical predictions with experiments invariably led to a perfect agreement. Physics is a human activity and from us inherits the habit to parade the successes and to hide difficulties and failures. Thus only silence surrounded the Sagnac effect (discovered in 1913) for which there is a veritable explanatory inability of the two relativistic theories, the attempts by Langevin, Post, Landau and Lifshitz notwithstanding. There are, furthermore, the half explanations of the aberration of starlight and of the clock paradox, phenomena for which the mathematical formalism of the theory can reproduce the observations at the price of twisting the meaning of symbols beyond rightfulness.

In reality the two relativistic theories are crammed with paradoxes: (a) The idea that the simultaneity of spatially separated events does not exist in nature and must therefore be established with a human convention; (b) The velocity of a light signal, considered equal for observers at rest and observers pursuing it at 0.99 c; (c) The contraction of moving objects and the retardation of moving clocks, phenomena for which the theory does not provide a description in terms of objectivity; (d) The existence of a discontinuity between the inertial reference systems and those endowed with a very small acceleration; (e) The propagations from the future towards the past, generated in the TSR by the possible existence of superluminal signals; (f) The empirically certain asymmetrical behavior of two clocks in relative motion in a theory waving the flag of relativism. Therefore the postulate of relativity receives a blow, and must somehow be negated. Perhaps “theory of relativity” is just a name, not to be taken too literally. The total relativism which the theory could seem to embody is now perceived to be an illusion. Not all is relative in relativity, because this theory contains also some features that are observer independent, then features which are absolute!
2. Relativistic paradoxes

Einstein’s theories had great success in explaining many known phenomena and in predicting new ones. Therefore they contain important advances of our knowledge of the physical world and belong forever to the history of the natural sciences, similarly to Newton’s mechanics and Maxwell’s electromagnetism. It is however difficult to believe that they are final forms of knowledge. On the contrary, the lesson to learn from epistemology (Popper, Lakatos, Kuhn) is about the conjectural, provisional, improvable nature of the physical theories of the XXth century.

In March 1949, answering his friend M. Solovine who had sent him an affective letter for the seventieth birthday, Einstein had written: “You imagine that I look backwards on the work of my life with calm satisfaction. But from nearby it looks very different. There is not a single concept of which I am convinced that it will resist firmly.” [2-1] Einstein did not hide the transitoriness of his creations. On April 4, 1955, he wrote the last paper of his life. It was a three pages long preface (in German) to a book celebrating the fiftieth anniversary of the theory of relativity. It ended with the following words: “The last, quick remarks must only demonstrate how far in my opinion we still are from possessing a conceptual basis of physics, on which we can somehow rely.” [2-2] In a way this is a declaration of failure, but one has to admire the ethical dimension of the great scientist who had devoted the superhuman efforts of a lifetime to the attempt of reaching the deepest truths of nature and now, arrived at the end, declares to posterity: “I did not succeed.”

The successes of the relativistic theories are very well known. The reciprocal convertibility of energy and mass, the effects of velocity and gravitation on the pace of clocks, the weight of light and the precession of planetary motions, provide only a partial summary of the great conquests of Einsteinian physics. Nevertheless, it would not be correct to conclude that every comparison of the theoretical predictions with experiments invariably led to a perfect agreement. Physics is a human activity and from us inherits the habit to parade the successes and to hide difficulties and failures. Thus only silence surrounded the Sagnac effect (discovered in 1913) for which there is a veritable explanatory inability of the two relativistic theories, the attempts by Langevin [2-3], Post [2-4], Landau and Lifshitz [2-5] notwithstanding. There are, furthermore, the half explanations of the aberration of star light and of the clock paradox, phenomena for which the mathematical formalism of the theory can reproduce the observations at the price of twisting the meaning of symbols beyond rightfulness.

One should never forget that behind the equations of a theory there is a huge qualitative structure made of empirical results, generalizations, hypotheses, philosophical choices, historical conditionings, personal tastes, conveniences. When one becomes aware of this reality and compares it with the little portrait of physics handed down by logical empiricism, which is worth less than a caricature, one easily understands that relativity, not only can present weak points side by side with its undeniable successes, but can also survive some failures. The correctness of the mathematical formalism is not enough to validate a scientific structure as coherent and not contradictory. I add that not even hundreds of physicists unconditionally favorable to a theory can warrant absence of unsolved problems, because much too often their thoughts are oriented since the university studies towards an acritical acceptance of the dominating theory.

In reality the two relativistic theories are crammed with paradoxes. Let us try to make a list, with no claim of completeness, limited to the TSR: 1. The idea that the simultaneity of spatially separated events does not exist in nature and must therefore be established with a human convention; 2. The relativity of simultaneity, according to which two events simultaneous for an observer in general are no more such for a different observer; 3. The
velocity of a light signal, considered equal for observers at rest and observers pursuing it with velocity \(0.99c\); 4. and 5. The contraction of moving objects and the retardation of moving clocks, phenomena for which the theory does not provide a description in terms of objectivity; 6. The hyper-deterministic block universe of relativity, fixing in the least details the future of every observer; 7. The conflict between the reciprocal transformability of mass and energy and the ideology of relativism, which declares all inertial observers perfectly equivalent so depriving energy of its full reality; 8. The existence of a discontinuity between the inertial reference systems and those endowed with a very small acceleration; 9. The propagations from the future towards the past, generated in the TSR by the possible existence of superluminal signals; 10. The asymmetrical ageing of the twins in relative motion in a theory waving the flag of relativism.

In section 18, after having established the validity of the IT, we will show that the previous paradoxes are fully overcome, so that they will disappear from the scientific debate as soon as the new IT will be accepted. The substitution of Einstein’s relativity principle with a weaker principle will also be one of the results of our present research.

Herbert Dingle, professor of History and Philosophy of Science in London, in the fifties and early sixties fought a battle against some features of the relativity theory, in particular against the asymmetrical ageing present in the clock paradox argument. He believed that the slowing down of moving clocks was pure fantasy. This idea has of course been demolished by direct experimental evidence, collected after his time. Nevertheless, his work has left posterity a rare jewel: the syllogism bearing his name. Given that syllogism is a technical model of perfect deduction, its consequences are absolutely necessary for any person accepting rational thinking in science. Dingle’s syllogism is the following [2-6]:

1. **(Main premise)** According to the postulate of relativity, if two bodies (for example two identical clocks) separate and reunite, there is no observable phenomenon that will show in an absolute sense that one rather than the other has moved.

2. **(Minor premise)** If upon reunion, one clock were retarded by a quantity depending on its relative motion, and the other not, that phenomenon would show that the first clock had moved (in an observer independent “absolute” sense) and not the second.

3. **(Conclusion)** Hence, if the postulate of relativity is true, the clocks must be retarded equally or not at all: in either case, their readings will concord upon reunion if they agreed at separation. If a difference between the two readings were to show up, the postulate of relativity cannot be true.

Today it can be said that the asymmetrical behaviour of the two clocks is empirically certain (muons in cosmic rays, experiment with the CERN muon storage ring, experiments with linear beams of unstable particles, Hafele and Keating experiment). Therefore, as a consequence of point 3. above, the postulate of relativity must somehow be negated. Actually, in recent times there are some authors who think that “theory of relativity” is just a name, not to be taken too literally. The total relativism which the theory could seem to embody is now perceived to be only an illusion. One can conclude that not all is relative in relativity, because this theory contains also some features that are observer independent, then features which are absolute! As Dingle wrote: “It should be obvious that if there is an absolute effect which is a function of velocity, then the velocity must be absolute. No manipulation of formulae or devising of ingenious experiments can alter that simple fact.” [2-7]

How is it possible that respected experts of relativistic physics believe that those listed above and numbered from 1 to 10 are not real paradoxes? The answer is not difficult and is based on what in Italian is called “buon senso” (literally: good sense). This expression is easily translated in all neo-Latin languages, but is absent in other languages. English speaking
authors use sometimes “common sense”, which carries however a very different idea because the common sense is that of the majority and the history of science teaches that in scientific matters the majority is rarely right.

Well, if good sense tells us that a certain prediction of a theory is unreasonable, there are two possibilities. Firstly, it is possible that the good sense misleads us, secondly that in the theory there are more or less explicit hypotheses contrary to the natural order of things giving its predictions an incorrect meaning. It is well known that many physicists and philosophers of science of the XXth century followed the fashion of declaring good sense obsolete, but we will show that the second road can easily be traveled over and allows one to get rid of all the paradoxes of relativity. Of course one could object that it is not a priori obvious that the paradoxes can be eliminated without spoiling the successes of the TSR. Nevertheless, it will be seen that the theory defended in this book, based on the replacement of the Lorentz by the “inertial” transformations and on a radical modification of the philosophical taste of the theory, not only explains all what the TSR does, but succeeds also where the latter does not. It explains the Sagnac effect, for example.

[2-1] Letter to M. Solovine
[2-2] [EF, p. xx]
[2-5] [LL, §§ 82-87]
Chapter 3

Relativism and the nature of energy

"Relativism" can be presented with an example. Two inertial reference systems are given, $S$, of the stationmaster and $S'$ of the passenger on the train. Let $v=0.6c$ be the train velocity, hence the Lorentz factor is $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 0.8$. It follows in the standard way that if the stationmaster sees a meter immobile in the train parallel to the rail and measures its length, he finds 80 cm. Similarly, if the passenger on the train sees a meter immobile in the railway station parallel to the rail, and measures its length he finds 80 cm. "My meter is longer than yours" will say the stationmaster, and the same words will be repeated by the passenger. Who is right? According to Einstein: they are both right and the contradiction is only apparent. An obvious generalization of the previous description is the basis of the relativism of the theory of relativity. Relativism is not necessary, it could be avoided. The TSR could be correct as a scientific theory and, at the same time, relativism could not hold. The strict tie (theory of relativity - relativism) was a free choice of Albert Einstein.

According to the TSR mass and energy are different forms of a unique reality, prediction corroborated by an enormous number of experiments of nuclear and subnuclear physics. The relativistic total energy $E$ (kinetic energy plus rest mass energy) of a particle having rest mass $m$ and velocity $v$ relative to a frame of reference $S$ is the first formula below, which holds, for the given particle, in all inertial systems $S, S', S'' \ldots$ provided one uses the particle velocity $u, u', u'' \ldots$ relative to each of them.

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$

From $E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$ to $E = \frac{mc^2(1 - \bar{u} \cdot \bar{V} / c^2)}{\sqrt{(1 - \bar{u} \cdot \bar{V} / c^2) - \bar{u}^2 / c^2}}$

Among all these different values, which is it the real value of energy? The TSR answers: all observers are equivalent, and since each of them attributes to the particle energy a different value, in the impossibility of choosing one value as "more true", one must conclude that a well defined value of energy does not exist. In this way energy is at once stripped by relativism of its most important property, that of having a well defined value.

For avoiding this unpleasant conclusion there is only one secure possibility, giving up the relativism that originates from the space-time symmetry of the Lorentz transformations. The retrieval of the objectivity of energy should rather aim at the inequivalence of the reference frames, inequivalence easily achieved with the inertial transformations (see chapter 7). Deduced from the inertial transformations the formula of the total energy $E$ (kinetic energy plus rest mass energy) of a particle having rest mass $m$ and velocity $u$ relative to a frame of reference $S$ coincides with the second formula above, where $\bar{V}$ is the velocity of $S$ relative to the privileged frame $S_0$. Notice that if $\bar{V} = 0$ this equation becomes the old one, which obviously gives also the true value of energy, by definition the value in $S_0$. 
An important paradox of the relativistic theory arises from the application of the idea of relativism to the physical quantities. When this is done they all seem to lose their concreteness and almost to vanish into nothingness, including the most fundamental one, energy. In the present section we will see why this happens and what has to be done to save their reality.

Let us start from the idea of relativism, which is best presented with an example. Two inertial reference systems are given, the system $S_0$ of the stationmaster and the system $S$ of the passenger on the train. Let $v = 0.6c$ be the train velocity, hence the Lorentz factor is \[ \frac{1 - v^2}{c^2} = 0.8. \] It follows in the standard way that if the stationmaster sees a meter immobile on the floor of the train parallel to the rail and measures its length, he finds 80 cm. It is equally clear that if the passenger on the train sees a meter immobile on the floor of the railway station parallel to the rail, and measures its length he finds 80 cm. They will both conclude that a meter moving at a speed 0.6c relative to their rest systems is actually 80 cm long. Who is right? According to Albert Einstein: they are both right in the same way. The latter statement (“they are both right in the same way”) is the basis of the relativism of the theory of relativity, but is not necessarily true. The TSR could be correct as a scientific theory and, at the same time, relativism could not hold. For example, Lorentz’s reformulation of the theory is experimentally indistinguishable from Einstein’s TSR while admitting the existence of ether and thus of a privileged inertial system. In Lorentz’s approach the opinions of stationmaster and passenger are not equivalent. For example, if $S_0$ were the privileged system, the stationmaster would be right and the passenger wrong. In general, the observer with smaller absolute velocity would give a better judgment about the true length of the meter.

The theory of relativity led to the conclusion that an arbitrary object, whose quantity of matter is measured by mass, and the motion of the same object, measured by energy, have the same properties and can be transformed into one another. This corresponds to a basic reality of energy, which shares all the properties of mass. For example: “If the theory corresponds to the facts, the radiation conveys inertia between the emitting and the absorbing bodies.” [3-1] Mass and energy have to be considered different forms of a unique reality. The reciprocal transformability of mass and energy has been confirmed in an enormous number of experiments of nuclear and subnuclear physics, so that it can now be considered an irreversible progress element of scientific knowledge. The mass-energy equivalence is expressed by the famous formula

$$E = mc^2$$

(3.1)

The unitary nature of energy and mass was so described in the book by Einstein and Infeld: “A further consequence of the (special) theory of relativity is the connection between mass and energy. Mass is energy and energy has mass. The two conservation laws of mass and energy are combined by the relativity theory into one, the conservation law of mass-energy.” [3-2]

The mass-energy equivalence had many consequences, for example it predicted a continuity between that form of energy diffused in space which is called “field” and the material sources generating it: “From the relativity theory we know that matter represents vast stores of energy and that energy represents matter. We cannot, in this way, distinguish qualitatively between mass and field, since the distinction between mass and energy is not a
qualitative one. We could therefore say: Matter is where the concentration of energy is great, field where the concentration of energy is small. But if this is the case, then the difference between matter and field is a quantitative rather than a qualitative one.” [3-3]

From the experimental point of view the mass-energy equivalence means that a material object can be transformed into pure motion (that is, into kinetic energy of other objects) and, viceversa, that it is possible to create matter at the expenses of motion. These transformations take place according to the rigorous laws of conservation of energy and momentum in absolutely concrete processes.

$$ P + P \rightarrow P + P + \pi^0 $$

It is possible to make two protons with high enough kinetic energy collide to produce in the final state the same two protons with identical properties (mass, electric charge, etc.) and, additionally, one or several new pieces of matter, for example $\pi$ mesons, which were born from nothingness during the collision. Rather, they seem to be born from nothingness to a person observing the phenomenon only superficially. Actually, if one compares the kinetic energies of the initial and final state one finds that exactly the quantity of kinetic energy has disappeared that is necessary to produce the new mass in the final state.

As we just saw, two colliding protons can give rise to a new physical state including two protons and a neutral $\pi$ meson. The meson $\pi$ is a quantum of nuclear forces and has a rest mass 264 times that of the electron. There are hundreds of examples of the same type: nucleons, mesons, gamma rays can replace the proton as the projectile particle, and in the final state many more combinations of the same particles (or of other particles: hyperons, strange mesons, for example) can appear. It has been checked that in every example the energy-momentum conservation laws are satisfied if Eq. (3.1) is used to express the energy content of the masses.

Inverse processes exist as well, in which energy is created at the expenses of mass. Of this type are the uranium fission reactions. In this way one sees how false was the belief of the past that matter can neither be created nor destroyed. In reality there is no law of conservation of matter: what is conserved under all circumstances is energy together with its vectorial daughter, the quantity of motion (momentum). These are the fundamental quantities of reality, whereas the stability of matter is pure appearance, due to the fact that we live in a low energy world. If the energy increases, matter can start to disappear! In fact at the center of the Sun there is a temperature of 15-20 million degrees, the kinetic energy of thermal agitation is correspondingly high and every second four million tons of matter are transformed into radiant energy.

It is rather obvious that the achievements of relativity on the just described mass-energy relationship would belong rather naturally to the philosophical field of realism. Positivism, however, did not disappear. On the contrary it extended its domination to the very notion of energy, as we will see next. Energy has all the right properties to be considered a kind of fundamental substance of the universe: it is indestructible, it enters in all dynamical processes and matter itself can be considered a localized form of energy. Naturally this “energetic materialism”, if possible, would be very different from the anti-atomistic energetism proposed by Ostwald towards the end of the XIXth century. However, according to the TSR energy has no fundamental role. Different inertial observers assign different velocities, and thus different energies to any given particle. The relativistic formula of the total energy $E$ (kinetic energy plus rest mass energy) of a particle having rest mass $m$ and velocity $u$ relative to a frame of reference $S$ is
where $c$ is the velocity of light, as usual. This formula holds, for the given particle, in all inertial systems $S$, $S'$, $S''$ ... provided one uses the particle velocity $u$, $u'$, $u''$, ... relative to each of them. If one asks which is the real value of energy, the TSR answers that all observers are equivalent, so that their calculations are all equally valid. And since each of them attributes to the particle energy a different value, in the impossibility of choosing one of these as “more true”, one is forced to conclude that a well defined value of energy does not exist. In this way energy, possible substratum of the universe, is at once stripped by relativism of its most important property, that of having an objectively well defined value.

In 1943 J. Jeans used a similar argument against the objectivity of forces. For him the essence of a physical explanation, at least classically, is that each particle of a system experiences a real and definite force. This force should be objective as regards both quantity and quality, so that its measure should always be the same, whatever means of measurement are employed to measure it - just as a real object must always weigh the same, whether it is weighed on a spring balance or on a weighing beam. But the TSR shows that if motions are attributed to forces, these forces will be differently estimated, as regards both quantity and quality, by observers who happen to be moving at different speeds, and furthermore that all their estimates have an equal claim to be considered right. “Thus - Jeans concludes - the supposed forces cannot have a real objective existence; they are seen to be mere mental constructs which we make for ourselves in our efforts to understand the workings of nature.” [3-4] Naturally for Jeans it was immediately possible to generalize his argument to all physical quantities: force, energy, momentum, and so on. With his words: “But the physical theory of relativity has now shown ... that electric and magnetic forces are not real at all; they are mere mental constructs of our own, resulting from our rather misguided efforts to understand the motions of the particles. It is the same with the Newtonian force of gravitation, and with energy, momentum and other concepts which were introduced to help us understand the activities of the world - all prove to be mere mental constructs, and do not even pass the test of objectivity. If the materialists are pressed to say how much of the world they now claim as material, their only possible answer would seem to be: Matter itself. Thus their whole philosophy is reduced to a tautology, for obviously matter must be material. But the fact that so much of what used to be thought to possess an objective physical existence now proves to consist only of subjective mental constructs must surely be counted a pronounced step in the direction of mentalism.” [3-5]

After such a striking conclusion it is no surprise that Jeans arrives to the most genuine philosophical idealism: “Today there is a wide measure of agreement, which on the physical side of science approaches almost to unanimity, that the stream of knowledge is heading towards a non-mechanical reality. The universe begins to look more like a great thought than like a great machine. Mind no longer appears as an accidental intruder into the realm of matter. We ought rather to hail it as the creator and governor of the realm of matter.” [3-6]

For avoiding these unpleasant conclusions there is only one secure possibility, giving up the philosophy of relativism that originates from the space-time symmetry of the Lorentz transformations, admittedly constituting the most natural interpretation of the Einsteinian theory. The retrieval of the objectivity of energy and of the other physical quantities should rather aim at the inequivalence of the different reference frames. But such lack of equivalence is easily achieved with the inertial transformations (see section 7 below), based on the existence of a privileged system, which give back to the mass-energy equivalence the great conceptual value of a substance leading to a possible unification of physics. Deduced from the
inertial transformations the formula of the total energy $E$ (kinetic energy plus rest mass energy) of a particle having rest mass $m$ and velocity $u$ relative to a frame of reference $S$ is

$$E = mc^2 \frac{1 - \frac{u \cdot \bar{v}}{c^2}}{\sqrt{(1 - \frac{\bar{u} \cdot \bar{v}}{c^2}) - \frac{u^2}{c^2}}}$$

(3.3)

where $\bar{v}$ is the velocity of $S$ relative to the privileged frame $S_0$ [3-7]. After overcoming the philosophy of relativism, energy can take up its fundamental role, its true value being the one calculated in the privileged isotropic inertial system. Notice that if $\vec{u} \cdot \vec{v} = 0$ Eq. (3.3) reduces to (3.2), the latter giving the true value of energy. After all, one remains with the impression that relativism is only an ideological element inserted in a physical reality (here that of energy) which is sound and perfectly capable to run the game of physics.

[3-1] A. Einstein, *Does the inertia of a body depend upon its energy content?* in [PR, p. 71].
[3-2] [EI, pp. 197-198]
[3-3] [EI, pp. 241-242]
[3-4] [JJ, p. 14]
[3-5] [JJ, p. 200]
[3-6] Quoted in [PF, p. 235]
Chapter 4

Einstein’s relativistic ether

“[...] in 1905, I was of the opinion that it was no longer allowed to speak about the ether in physics. This opinion, however, was too radical [...]. It does remain allowed, as always, to introduce a medium filling all space and to assume that the electromagnetic fields (and matter as well) are its states.” Albert Einstein.

Strictly speaking, the abolishment of ether cannot be considered a necessary consequence of Einstein’s relativism. Relativism demands only that the description of the physical reality be the same in all inertial reference frames and this can be achieved also with an ether deprived of mobility: “More careful reflection teaches us, however, that this denial of the existence of the ether is not demanded by the special principle of relativity. We may assume the existence of an ether; only we must give up ascribing a definite state of motion to it, i.e., we must by abstraction take away from it the last mechanical characteristic that Lorentz had still left it.”

Thus Einstein was pushed to admit the limited horizon of his research on the physics of space and time: “It would have been more correct if I had limited myself, in my earlier publications, to emphasise only the nonexistence of an ether velocity, instead of arguing the total nonexistence of the ether, for I can see that with the word ether we say nothing else than that space has to be viewed as a carrier of physical qualities.” Space is indeed such a carrier: it gives us the possibility to identify the inertial reference systems, or, which is the same, the inertial forces in the accelerated systems. “On the other hand there is a weighty argument to be adduced in favor of the ether hypothesis. To deny the existence of the ether means, in the last analysis, denying all physical properties to empty space. But such a view is inconsistent with the fundamental facts of mechanics.”

Summarizing, we can say that according to the TGR space is equipped with physical properties; also in this sense an ether exists. According to the TGR space without ether is unthinkable, as in such a space not only the propagation of light would not take place, but also there would be no possibility of existence for clocks and rodes, so that also no spatiotemporal distances "in the sense of physics.”
4. Einstein’s relativistic ether

In the 1905 paper introducing the theory of special relativity Einstein wrote that the hypothesis of a luminiferous ether could be considered superfluous, given that the new theory needed neither an absolutely stationary space endowed with particular properties, nor a medium in which electromagnetic processes, such as the propagation of light, could take place. Einstein started to reconsider the whole question of the ether in the years of his explicit transition from positivism to realism (1916-1924). At this time he admitted that it was still possible to think ether as existing, even if only to designate particular properties of space. The reasoning which promoted the ether idea from superfluous to admissible was more or less the following.

If every ray of light propagates in the vacuum with velocity $c$ relative to the inertial system $K$, we must imagine this luminiferous ether everywhere at rest with respect to $K$. But if the laws of propagation of light relative to the different inertial system $K'$ (moving with respect to $K$) are the same as relative to $K$, we must with the same right accept the existence of a luminiferous ether at rest with respect to $K'$. The standpoint of the 1905 formulation of the TSR was that it is absurd to accept that ether is at rest at the same time in both systems and that one must give up introducing it. After 1916 Einstein modified his position and assumed that ether is somehow at rest both with respect to $K$ and $K'$, that is to say, given the arbitrariness of $K$ and $K'$, at rest at the same time with respect to all inertial frames. It was certainly a very unusual idea to deprive a physical entity of the right to be seen in motion, but that was Einstein’s choice. He so described the situation:

“[...] in 1905, I was of the opinion that it was no longer allowed to speak about the ether in physics. This opinion, however, was too radical [...] . It does remain allowed, as always, to introduce a medium filling all space and to assume that the electromagnetic fields (and matter as well) are its states. But, it is not allowed to attribute to this medium a state of motion in each point, in analogy to ponderable matter. This ether may not be conceived as consisting of particles that can be individually tracked in time.” [4-1]

The abolishment of the ether cannot be considered a necessary consequence of Einstein’s relativism. This philosophy, embodied in the principle of relativity, demands only that the description of the physical reality be the same in all inertial reference frames and this can be achieved also with an ether deprived of mobility: “More careful reflection teaches us, however, that this denial of the existence of the ether is not demanded by the special principle of relativity. We may assume the existence of an ether; only we must give up ascribing a definite state of motion to it, i.e., we must by abstraction take away from it the last mechanical characteristic that Lorentz had still left it.” [4-2] If one considers pointlike particles one is bound to conclude that motion is always possible, for a particle has well defined values of the space-time coordinates and the Lorentz transformations can be applied to produce motion from rest. However: “Extended physical objects can be imagined to which the idea of motion cannot be applied. They are not to be thought of as consisting of particles that allow themselves to be separately tracked through time. In Minkowski’s idiom this is expressed as follows: Not every extended conformation in the four-dimensional world can be regarded as composed of lines of Universe.” [4-3] In this way the ether is postulated to be devoid of motion. Obviously this means that also the notion of “motionlessness” cannot be applied to it, at least because immobility is a particular case of motion with zero velocity. Thus Einstein writes: “As to the mechanical nature of the Lorentzian ether, it may be said of it, in a somewhat playful spirit, that immobility is the only mechanical property of which it has not been deprived by H.A. Lorentz. It may be added that the whole change in the
conception of the ether, which the special theory of relativity brought about, consisted of taking away from the ether its last mechanical quality, namely, its immobility." [4-4]

Thus Einstein was pushed to admit the limited horizon of his research on the physics of space and time: “It would have been more correct if I had limited myself, in my earlier publications, to emphasise only the nonexistence of an ether velocity, instead of arguing the total nonexistence of the ether, for I can see that with the word ether we say nothing else than that space has to be viewed as a carrier of physical qualities.” [4-5] And space is indeed such a carrier: such as the possibility to introduce in every point of space well defined inertial reference systems, or, which is the same, the inertial forces in the accelerated systems. “On the other hand there is a weighty argument to be adduced in favor of the ether hypothesis. To deny the existence of the ether means, in the last analysis, denying all physical properties to empty space. But such a view is inconsistent with the fundamental facts of mechanics.” [4-6]

Therefore one can say that “physical space” and “ether” are only different terms for indicating the same reality. Furthermore, fields are physical states of space. If no particular state of motion can be attributed to the ether, there does not seem to be any reason for introducing ether as an entity of a special type alongside of space. Naturally it is not forbidden to use the word ether, but only to express the physical properties of space. The word ether changed its meaning many times in the development of science. Around 1920, it no longer stood for a medium built up of particles. Its story, by no means finished, was to be continued by the relativity theory. In conclusion, “Summarizing, we can say that according to the theory of general relativity space is equipped with physical properties; also in this sense an ether exists. According to the general theory of relativity space without ether is unthinkable, as in such a space not only the propagation of light would not take place, but also there would be no possibility of existence for clocks and rodes, so that also no spatiotemporal distances "in the sense of physics."” [4-7]

Einstein’s new ether was introduced in connection both with the TSR and with the GTR. The second case is more interesting physically, as: “According to general relativity, the concept of space detached from any physical content does not exist. The physical reality of space is represented by a field whose components are continuous functions of four independent variables - the coordinates of space and time. It is just this particular kind of dependence that expresses the spatial character of physical reality.” [4-8]

The gravitational field as described by the potentials, continuous functions of the coordinates of space and time: “No space [can be conceived] without gravitational potentials; for these give it its metrical properties without which it is not thinkable at all. The existence of the gravitational field is directly bound up with the existence of space.” [4-9] Depending on the different nearby masses the ether can be found in different states: “The ether of the general theory of relativity therefore differs from that of classical mechanics or the special theory of relativity, in so far as it is not ‘absolute’, but is determined in its locally variable properties by ponderable matter.” [4-10]

With the mathematics of the general theory of relativity it is possible to implement a continuous transition from STR to GTR, thus incorporating the laws of nature, already known from STR, into the broader framework of GTR: “The real is conceived as a four-dimensional continuum with a unitary structure of a definite kind (metric and direction). The laws are differential equations, which the structure mentioned satisfies, namely, the fields which appear as gravitation and electromagnetism. The material particles are positions of high density without singularity. We may summarize in symbolical language. Space, brought to light by the corporeal object, made a physical reality by Newton, has in the last few decades swallowed ether and time and seems about to swallow also the field and the corpuscles, so that it remains as the sole medium of reality.” [4-11]
The comparison of Einstein’s views about ether with Poincaré’s views has recently been made by Y. Pierseaux [4-12], who wrote: “Poincaré has a relativistic ether and has not abolished it (as Einstein did), because it remains the relativistic definition of the state of rest: when one sets the ether at rest in one system by definition (spheres or true time), it is not at rest in the other (ellipsoids or local time).” [4-13] Clearly, Poincaré does not use the Lorentz transformations in the work described in the above quotation. He rather deduces the ellipsoidal shape of the light wave directly from the principle of contraction of length. But one cannot deduce two contradictory consequences from the same theory, so that Poincaré in that moment had in mind a different theory incorporating the Lorentz contraction. This is almost certainly the theory of the inertial transformations!

Thinking one last time of those past events, one realizes that it must have been difficult for Albert Einstein to resist Lorentz’s pressure in favour of ether. One can say that the ether at rest in all inertial frames was his way to concede space to Lorentz while defending relativism. It was not a great idea of the type he had so many times during his life and today it remains half forgotten. It is interesting philosophically as an attempt to put together realism (with ether) and relativism.

[4-1] [MM, § 13]
[4-2] [AR, p. 9]
[4-3] [AR, p. 10]
[4-4] [AR, p. 7]
[4-6] [AR, p. 11]
[4-7] [AR, p. 15]
[4-8] [MR, p. 348].
[4-9] [AR, pp. 13-14]
[4-10] [PV, p. 18]
[4-13] [EP, p. 122]
5. **Is relativistic simultaneity conventional?**

In some cases Einstein seemed to admit a conventional nature of the postulate concerning the one way velocity of light in the relativistic theory. In his fundamental article of 1905 on the theory of special relativity (TSR) he wrote: “We have so far defined only an “A time” and a “B time”. We have not defined a common “time” for A and B, for the latter cannot be defined at all unless we establish by definition that the “time” required to light to travel from A to B equals the “time” it requires to travel from B to A.” [5-1].

Besides establishing by definition some properties of time, this statement is remarkable for two further reasons showing the positivistic inclinations of the founder of relativity. Firstly, it accepts Poincaré's idea that the speed of light is not measurable and can then only be defined; secondly, the word time, appearing five times, is always in quotes, as if it were a dangerous conception. The conventionality of the velocity of light was restated in 1916 when Einstein wrote about the midpoint $M$ of a segment $AB$ whose extreme points are struck “simultaneously” by two strokes of lightning: “that light requires the same time to traverse the path $AM$ ... as the path $BM$ [$M$ being the midpoint of the line $AB$] is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own free will.” [5-2]  

Before the birth of relativity the problem was how to synchronize clocks in different positions. A method for synchronizing distant clocks coming to every beginner’s mind does not work: synchronize them when they are near and carry them in the points where they are needed. It does not work because we know that transport, that is the fact itself of possessing a velocity, modifies any periodic motion that one might use for measuring time. But we do not know with respect to which inertial system the clock velocity should be considered, and at this stage of the game we are anyway unable to measure it. Given this situation, Poincaré and Einstein decided that the “synchronization” of clocks could be achieved following criteria of any type, provided only they led to a non ambiguous identification of events. The author of relativity made the simplest choice, assuming that the speed of light has the same value in all directions in all inertial frames.  

In practice, the relativistic synchronization is obtained as follows. Suppose that two identical clocks are at a distance $L$ from one another. A pulse of light starts from $A$ towards $B$ when the clock in $A$ marks time zero; the clock in $B$ is set at time $L/c$ when the pulse arrives there, as if the velocity of light from $A$ to $B$ were known to be $c$. At this point the two clocks are synchronized and it becomes possible to use them for one way velocity measurements. Similar events repeat themselves similarly: sometimes this natural assumption is called “homogeneity
of time.” Therefore, if a new pulse of light starts from $A$ towards $B$ when the clock in $A$ marks time $\Delta$, the clock in $B$ will mark time $\Delta + L/c$ when the pulse arrives there. The velocity of light propagating from $A$ to $B$ is, by definition,

$$\text{velocity} = \frac{\text{space}}{\text{time}} = \frac{L}{(\Delta + L/c - \Delta)c}.$$ 

From synchronization to relativistic simultaneity the step is short. Two instantaneous point-like events in $P$ and $Q$ at times $t_p$ and $t_B$ (as marked by the respective synchronized local clocks) are simultaneous by definition if $t_p = t_B$. Naturally a good positivist does not wonder whether the two events are really simultaneous: for him only human manipulations matter and it does not make any sense to think in terms of an objectivity of time. Thus the notion of relativistic simultaneity depends on human stipulations and not on properties of nature.

Here it is useful to stress that the conventional nature of relativistic clock synchronization - and then of the relativistic simultaneity of distant events - if real, would open very interesting perspectives. Let us see why. In general, time could be different in two different inertial reference systems $S_0(x_0, y_0, z_0, t_0)$ and $S(x, y, z, t)$, and the “delay” $t - t_0$ (positive, null or negative) of $t$ over $t_0$ could depend not only on the time $t_0$, but also on the considered geometrical point. This happens in the TSR, as the Lorentz transformation of time contains also a space coordinate. In other words (and more generally) the time $t_0$ marked by a clock of $S_0$ can depend also on the coordinates $x, y, z$ of the point at which the clock is positioned, at least until one finds reasons for the contrary (I found them, see the chapters with $e_1 = 0$ in the title).

Discussing this problem H. Reichenbach (1925) examined the following situation: in the system $S$ a flash of light leaves point $A$ at time $t_1$, is reflected backwards by a mirror placed in point $B$ at time $t_2$ and finally returns in $A$ at time $t_3$. Naturally $t_1$ and $t_3$ are marked by a clock near $A$, while $t_2$ is marked by a different clock near $B$. The problem is how to synchronize the two clocks with one another. In the TSR one assumes that the velocity of light on the two way path $A-B-A$ is the same as in the one way path $A-B$, so that

$$t_2 - t_1 = \frac{1}{2}(t_3 - t_1) \quad (5.1)$$

This formula defines the time $t_2$ of the $B$ clock in terms of the times $t_1$ and $t_3$ of the $A$ clock. It is the choice (5.1), which determines the presence of $x$ in the (Lorentz) transformation of time. Reichenbach commented that Eq. (5.1) is
essential in the TSR, but it is not epistemologically necessary. A different rule of the form
\[ t_2 - t_1 = \varepsilon (t_3 - t_1) \]  
(5.2)
with any \( 0 < \varepsilon < 1 \) would likewise be adequate and could not be considered false.

He added: "If the special theory of relativity prefers the first definition, i.e., sets \( \varepsilon \) equal to 1/2, it does so on the ground that this definition leads to simpler relations." [5-3]

If \( L \) is the \( AB \) distance, it trivially follows from the definition (5.2) of the Reichenbach parameter
\[ \varepsilon = \frac{1}{2} \frac{(t_2 - t_i)/L}{(t_3 - t_i)/2L} = \frac{1}{2} \frac{c_1(\theta)}{c_2} \]  
(5.3)

where \( c_1(\theta) \) and \( c_2 = c \) are the one way and two way velocity of light, respectively. That the former depends on the light propagation direction via the angle \( \theta \) and the latter does not is something that will be seen later. Reichenbach did not anticipate the dependence of \( \varepsilon \) on \( \theta \) resulting from

In 1979 Max Jammer discussed Reichenbach's \( \varepsilon \) coefficient stressing that one of the most fundamental ideas underlying the conceptual edifice of relativity is the conventionality of intrasystemic distant simultaneity. He added: "The "thesis of the conventionality of intrasystemic distant simultaneity" ... consists in the statement that the numerical value of \( \varepsilon \) need not necessarily be 1/2, but may be any number in the open interval between 0 and 1, i.e. \( 0 < \varepsilon < 1 \), without ever leading to any conflict with experience." [5-4]

It was also in the late seventies when the question of clock synchronization was discussed in three important papers by Mansouri and Sexl [5-5]. These were truly fundamental papers containing for the first time a detailed treatment of different synchronization methods and a physical discussion of the Tangherlni transformations. Their conclusion has influenced the whole subsequent debate on the foundations of relativistic physics: “Thus the much debated question … concerning the empirical equivalence of special relativity and an ether theory taking into account time dilatation and length contraction but maintaining absolute simultaneity can be answered affirmatively.”

After Jammer and Mansouri and Sexl theoretical research tried to give a practical confirmation to the conventionality conjecture. It did so almost twenty years later, a time lag unfortunately typical for this type of research. I started working at the relativistic questions around 1994. The practical confirmation was only partial and was accompanied by several unexpected results. Nevertheless, the
conclusion was reached that there is an important logical space for a nonrelativistic value of $\varepsilon$, that is, in the final analysis, for a theory alternative to the TSR! After a century of relativism one can finally open the doors to a different physics without conflicting with the enormous bulk of experimental results accumulated to date.

If one considers the general ideas about space and time that will be discussed in the seventh chapter one can anticipate that there is an important difference between clock synchronization in $S_0$ and $S$. In the case of $S_0$ it is dictated by a physical condition, the belief that an inertial frame exists relative to which the velocity of light is objectively the same in all directions, while in the case of $S$ it is instead considered a free choice to be based on conveniency. Of course the TSR makes the simplest choice by postulating the invariance of the one way velocity of light. Being a convention, this is an assumption of a particular type, grounded on the belief that one way velocities are not measurable and that one of them can be freely taken to have any (finite) desirable value. I repeat, these are general ideas, but they do not represent at all the thesis defended in this book.

In a recent book, entirely devoted to the notion of simultaneity, Jammer stresses that Einstein's 1905 analysis of the concept of distant simultaneity inaugurated the conceptual revolution of modern physics, and adds: "If, as mentioned above, the concept of distant simultaneity is a fundamental ingredient in the logical structure of the theory of relativity, but is in reality nothing but a convention, the question naturally arises of whether this does not imply that the whole theory of relativity and with it a major part of modern physics are merely fictions devoid of any actual content. A positive answer to this question would have disastrous consequences for the philosophical understanding and epistemological status of physics and with it of the whole of modern science." [5-6]

Substitute “disastrous” with “wonderful” and Jammer's words express also my personal opinion. I devoted ten years of work to the practical confirmation of the Reichenbach-Jammer intuition that simultaneity is conventional. The confirmation came out only in part, and with some important “surprises” (unexpected problems and solutions). Of course the whole matter will be discussed in detail in the remaining part of the book. Here I can briefly anticipate the conclusions obtained from the experiments examined (there are many more not yet analysed). My conclusions are as follows:

E1. **Set of experiments insensitive to clock synchronization**, that is set of experiments with outcome predicted equally well by the theory of the inertial
transformations (TIT) and the TSR. The set includes the experiments made by: Michelson-Morley, Kennedy-Thorndike, Maiorana, Ives-Stilwell, Fizeau, the TAI (International Atomic Time), ...

E2. **Set of experiments preferring the TIT over the TSR/TGR** that is of experiments predicted correctly by the TIT, but not finding a rational explanation from the TSR and/or the TGR. In this set are included: Sagnac effect, zero acceleration discontinuity of the velocity of light, aberration of the starlight, block universe paradox, ...

E3. **Set of experiments preferring the TSR/TGR over the TIT**: empty set.

The situation seems to be clear: the TIT explains all the examined experiments, while the two relativistic theories have serious problems with the experiments of the second set. In this way we will see that there is an important logical space for a theory alternative to the TSR.

An attempt to refute the conventional nature of the simultaneity of the Theory of Special Relativity and to defend somehow its objectivity has been made by the philosopher M. Friedman [5-7]. Generally speaking it could seem that my position is similar to his, but actually his point of view is weak as based only on the structural simplicity of Minkowski’s space, that nobody denies. The real question is to see whether with some slight mathematical complication one can obtain a better description of reality. And as far as I can see the answer has to be positive.

Concerning the question in the title of this chapter, in view of the results reviewed in the present book, which hopefully should constitute a serious blow to conventionalism, one can say that the simultaneity adopted in the TSR, more than conventional, is arbitrary and, it turns out, not correct. My recent research has shown that the arbitrariness of relativistic simultaneity opens a logical space to a different theory (the theory of the inertial transformations) that agrees with experiments even better than the TSR.
[5-1] [PR, p. 40]
[5-2] [SG, p. 18]
[5-3] [HR, p. 127]
[5-4] [TF, p. 205]
[5-6] [MJ, pp. 5-6]
[5-7] See the book [ST]
Chapter 6

The basic empirical evidence

In relativistic physics there are two pieces of experimental knowledge so solid that one can use them as basis for new theoretical developments: the invariance of the two way velocity of light, \( c \), and the retardation of moving clocks.

The XXth century physicists and philosophers of science were convinced that it is impossible to measure one way velocities: the propagations of atoms, nuclei, cosmic rays, light pulses, and so on, should remain forever a numerical mystery. Years before the formulation of the TSR, Poincaré had discussed the independence of the velocity of light of its propagation direction and claimed the impossibility to measure the one way velocity. The problem is typically relativistic: in classical physics nobody had ever thought that a similar obstacle could be met, probably because in classical physics no upper limit for velocities exists, so that in principle all clocks of any inertial system can be instantaneously informed of the time marked by some central clock. In concrete calculations classical physics can be applied when velocities of the relevant objects are much smaller than that of light; in this sense we can say that we move in space at a (one way) velocity of 2-300 km/sec (about 0.1% of the speed of light) as we take part, with the solar system, in the rotation of our spiral galaxy, the Milky Way.

One of the most precise measurements of \( c \) has an error of only 20 cm/sec. Thus \( c \) is known with a precision of \( 10^{-9} \), a thousand times better than needed for detecting second order effects due to the Earth motion. Yet, before and after 1978 one always found the same value within errors, and no dependence on the propagation direction. We conclude that \( c \) is invariant.

The retardation of moving clocks is nowadays very well known. In a 1977 experiment the lifetimes of positive and negative muons were measured at CERN. Muons with a velocity of 0.9994 \( c \), corresponding to \( R=0.0346 \), were circling in a storage ring with diameter 14 \( m \), with a centripetal acceleration of \( 10^{18} \) \( g \). The lifetime \( \tau \) was found in excellent agreement with the formula \( \tau = R \tau_0 \) where \( \tau_0 \) is the laboratory lifetime of muons circling in the ring. The lesson concerns the transformation of time: the laboratory time interval \( \tau_0 \) between two events in the same position of the moving system (injection and decay of the muon) is observed to be dilated according to \( \tau = R \tau_0 \) if compared with the corresponding \( \tau \) measured by the observer in motion who sees the muons at rest. The huge acceleration has instead no effect whatsoever on the lifetime. Muons in the storage ring decay exactly like muons of a rectilinear beam if velocity is the same.
6. The basic empirical evidence

We move in space at a speed of 2-300 km/sec (about 0.1% of the speed of light) as we take part, with the Sun, to the rotation of our spiral galaxy, the Milky Way. According to the Galilei-Newton physics the velocity of light relative to a terrestrial laboratory should depend on the propagation direction. In fact, let \( \tilde{c} \) be the velocity of a punctiform light signal with respect to the privileged system \( S_0 \). If \( \tilde{c}' \) is the velocity of the same signal with respect to a terrestrial laboratory, moving in with velocity \( \vec{v} \), one should have \( \tilde{c}' = \tilde{c} - \vec{v} \). Therefore \( c' \) should vary from \( c - v \) to \( c + v \) when the light propagation direction changes from parallel to antiparallel to \( \vec{v} \).

It could be thought that these effects of the first order in \( v/c \) are easily observable. One should however recall that even before the birth of the TSR Poincaré had argued the impossibility to measure the velocity of a light signal propagating between two different points. To understand the reasons of this conclusion let us consider a light pulse traveling from \( A \) to \( B \). If in \( B \) there is a mirror reflecting the signal backwards, it is enough to have a clock near \( A \) measuring the times \( t_1 \) and \( t_3 \) of start and return. The speed of the pulse is then given by its definition as length over time:

\[
c^2 = \frac{2d_{AB}}{t_3 - t_1}
\]

(6.1)

where \( d_{AB} \) is the \( A - B \) distance that can be measured in the standard way using a rigid rod. However, the so defined \( c_2 \) is a two way velocity and it is possible that the signal velocity from \( B - A \) be different. For measuring the latter, two synchronized clocks would be needed, one near \( A \) and the other one near \( B \). Unfortunately even today there is no agreement on how to synchronize distant clocks.

Years before the formulation of the TSR, Poincaré discussed the independence of the velocity of light of its propagation direction and stated: "That light has a constant velocity and in particular that its velocity is the same in all directions ... is a postulate without which it would be impossible to start any measurement of this velocity. It will always be impossible to verify directly this postulate with experiments." [6-1] Agreeing on the impossibility to measure the one way velocities, Einstein decided to solve the problem by decree, assuming the
invariance of the velocity of light: the second postulate of the TSR. In fact, he described this hypothesis not as a property of nature but as the stipulation among men.

One of the most precise measurements of \( c_2 \) was performed in 1978 by a British group [6-2] and gave the result:

\[
c_2 = (299,792,4588 \pm 0,0002) \text{ km/sec} \tag{6.2}
\]

confirmed by subsequent measurements [6-3]. Thus \( c_2 \) is known with a 10\(^{-9}\) precision, a thousand times better than needed for detecting the second order effects due to the Earth motion. Yet, before and after 1978 one always found the same value within errors, and no dependence on the propagation direction was observed, in agreement with the more indirect experiments (such as the Michelson-Morley experiment) that tried to detect the existence of the privileged reference system. Thus we have our first fundamental conclusion:

**R1.** The two way velocity of light is empirically invariant, meaning that it is independent of the propagation direction, of the time at which it is measured, and of the inertial reference system with respect to which it is considered.

With their 1887 experiment Michelson and Morley concluded that no shifts of the interference figures existed due to the Earth motion. To explain this result Fitzgerald and independently Lorentz supposed that the motion of an object through the ether with velocity \( v \) generated its shortening in the direction of velocity by the factor

\[
R = \sqrt{1 - v^2 / c^2} \tag{6.3}
\]

The idea of a contraction due to motion was not so strange as it might seem. In fact, using classical physics, Lorentz was able to prove that the motion of an electric charge through ether modifies its electric field by squeezing it towards a plane perpendicular to the direction of motion, and that the degree of squeezing increases with velocity [6-4]. Thus an electron bound to a moving proton no longer forms a regular atom, but the internal motion takes place on an orbit squeezed similarly to the field. Furthermore, the period of the electronic motion is modified, actually increased for the observer watching the atom move. One must therefore expect that every object (made of atoms) be shortened in its dimension parallel to velocity, and that in every moving clock the pace of advancement of the hands be slowed down.
In 1900 Larmor [6-5] considered a system "composed of two electrons of opposite charge" (one would say today: composed of an electron-positron pair), neglected irradiation, and assumed circular orbits round the common centre of mass of the two particles. Assuming also that the whole system was in motion through ether, he proved that the velocity dependent deformation of the electric fields generated in the bound system exactly the contraction postulated by Fitzgerald and Lorentz. Furthermore Larmor found that the orbital period was necessarily increased by \( R^{-1} \), where \( R \) is given by (6.3). This was the first correct formulation of the idea of a velocity dependent retardation of clocks.

\[ \mu \]

Figure 1. In the CERN storage ring unstable particles ("muons") circulated with a speed smaller than that of light by only six parts over ten thousand. It was observed that muons disintegrated after a lifetime about 29 times larger than for muons at rest.

The slowing down of moving clocks is nowadays experimentally very well ascertained. An experiment was performed in 1977 [6-6] when the lifetimes of positive and negative muons were measured at the CERN muon storage ring. Muons with a velocity of \( v = 0.9994 \, c \), corresponding to a factor \( R^{-1} = 28.87 \), were circling in a ring with diameter 14 m, with a centripetal acceleration equal to \( 10^{18} \, g \). The lifetime was measured and found in excellent agreement with the formula

\[ \tau_0 = \frac{\tau}{R} \]

where:

- \( \tau_0 \) is the lifetime of circling muons as observed from the laboratory;
- \( \tau \) is the lifetime of the same muons as seen by observers at rest wrt the muons.

In practice for \( \tau \) one adopts the ordinary value of the lifetime observed on muons almost at the zero energy limit in the laboratory:
\[ \tau = \left( 2.19703 \pm 0.00004 \right) \cdot 10^{-6} \, s \]

The lesson learnt from this experiment concerns the transformation of time: the laboratory time interval \( \tau_0 \) between two events taking place in the same position of the moving system (injection and decay of the muon) is observed to be dilated according to \( \tau_0 = \tau / R \) if compared with the corresponding time interval \( \tau \) measured by the observer who sees the muons at rest. We adopted here an empirical point of view about the famous “time dilatation” phenomenon, but of course the idea that all clocks in motion might go slow was born for the first time in Einstein’s fertile mind in 1905 when the TSR was formulated [6-7].

The independence of the lifetime on the huge gravitational forces acting on the muons is very remarkable. The situation must be different from what is observed on atomic clocks transported in the gravitational field of the Earth where even a change of altitude of 1 \( m \) of the system (airplane, satellite) carrying the clock results in an observable modification of the atomic clock rate. In the case of the CERN muons, instead, it is very likely that one could build a muon storage ring twice as large in linear dimensions, but capable of transporting muons with the same velocity of 0.9994\( c \). Such muons would feel inertial forces considerably different from those of the actual storage ring, but we know enough to be able to anticipate that nothing would change in their behavior: they would not feel the presence of any gravitational field. The whole matter will be discussed more deeply in the chapter devoted to the differential retardation of separating and reuniting clocks.

Particle physics experiments with accelerators and cosmic rays always gave results consistent with Eq. (6.4) in situations in which no acceleration was present. The first experiment of this type was a 1941 research by Rossi and Hall [6-8]. Previous experiments made in flight and on the ground on cosmic ray muons had revealed an unexpected phenomenon. The number of muons propagating vertically was more strongly reduced by a layer of air than by a dense absorber equivalent to the air layer with regard to ionization losses. Furthermore, the difference in stopping power between air and dense materials increased when the muon momentum was decreased. Clearly, one could not modify the momentum spectrum of cosmic rays, but it was possible to study separately different groups of muons.

The anomalous absorption in air was interpreted on the hypothesis that muons disintegrate spontaneously with a lifetime of the order of a few microseconds. According to this assumption, a considerable fraction of the muon beam will disappear by disintegration while traveling in the atmosphere. No appreciable number of muons, however, will disintegrate within a dense absorber,
The time required for the traversal of such an absorber is very short compared with the lifetime of muons. The Rossi-Hall experiment verified this aspect of the problem and found the lifetime

\[ \tau_0 = 2.4 \times 10^{-6} \text{ sec} \]

This experiment, however, had been made with the purpose of checking one of the most fundamental ideas of relativistic physics, the dependence of lifetime on velocity according to (6.4). Also this vital part ended with a success and with the full confirmation of the relativistic effects.

In the 1972 experiment with macroscopic clocks by Hafele and Keating [6-9] six accurately synchronized Cesium atomic clocks were used, and:

1) two were carried by ordinary commercial jets in a full eastbound tour around the planet;

2) other two were carried by ordinary commercial jets in a full westbound tour around the planet;

3) the last two remained on the ground.

It was observed that with respect to the latter clocks, those on board the westbound trip had undergone a loss of \((59 \pm 10) \text{ ns}\), while the clocks on the eastbound trip had undergone an advancement of \((273 \pm 7) \text{ ns}\). These results were in excellent agreement with the usual formula

\[ \tau_0 = \frac{\tau}{R} \]  

if:

a) one used three different factors \( R^{-1} \) for the three pairs of clocks. The largest (smallest) factor was that of clocks that traveled eastward (westward) for which the Earth rotation velocity added to (subtracted from) the jet velocity. That is, it was necessary to refer movements not to the Earth surface, but to a reference frame with origin in the Earth center and axes oriented toward fixed directions of the sky;
b) one kept into account the effect of the Earth gravitational field that varies with altitude and therefore modifies the rates of traveling clocks differently from those left on the ground.

The results of the Hafele-Keating experiment have been confirmed by the GPS (Global Positioning System) system of satellites [6-10]. This system consists of a network of 24 satellites in roughly 12-hour orbits, each carrying atomic clocks on board. The orbital radius of the satellites is about four Earth radii. The orbits are nearly circular, with eccentricities of less than 1%. Orbital inclinations to the Earth equator are about 55°. The satellites have orbital speeds of about 3.9 km/sec in a frame centered on the Earth and not rotating with respect to the stars. Every satellite has on board four atomic clocks marking time with an error of a few ns/day. From every point of the Earth surface at least four satellites are visible at any time. Conceived for military aims, the GPS was subsequently used for telecommunications, satellite navigation, meteorology.

The theory of general relativity predicts that clocks in a stronger gravitational field will tick at a slower rate. Thus the atomic clocks on board the satellites at GPS orbital altitudes will tick faster by about 45.900 ns/day because they are in a weaker gravitational field than atomic clocks on the Earth surface. The velocity effect predicts that atomic clocks moving at GPS orbital speeds will tick slower by about 7.200 ns/day than stationary ground clocks. Therefore the global prediction is a gain of about 38.700 ns/day. Rather than having clocks with such large rate differences, the satellite clocks were reset in rate before launch (slowing them down by 38.700 ns/day) to compensate for the predicted effects. The very rich data show that the on board atomic clock rates do indeed agree with ground clock rates to the predicted extent. Thus the theoretical predictions are confirmed, in particular the slowdown of the clock rate due to the orbital velocity.

We can then write the second basic empirical result:

R2 Clock retardation takes place in accordance with Eq. (6.4) with \( R \) given by (6.3) when clocks move with velocity \( v \) with respect to the privileged system \( S_0 \), which, in practice will be an inertial system having origin coincident with the Earth center and axes pointing to fixed directions of the sky.

We left in vagueness the question of the reference frame with respect to which \( v \) should be calculated. In the next chapter we will take R1. and R2. as fundamental empirical facts and eliminate all residual vagueness by making a new assumption: its validity will be corroborated by the success of the ensuing theory.
Up to this point I have presented the line of thought on which this book and more generally my research have been based. It could be considered an “almost orthodox” interpretation of the experimental evidence. It is better to add, however, that heresy exists also here and with a fair probability of being finally correct. The conviction that something is wrong with the standard interpretation of the experimental evidence has been forcefully expressed by Cahill: “… it is now known that numerous experiments, beginning with the Michelson-Morley experiment of 1887, have always shown that the postulates themselves are false, namely that there is a detectable local preferred frame of reference.” [6–11]

[6-5]  See the book [AM].
[6-10]  T. van Flandern, in: [OQ, pp. 81-90].
Chapter 7

The new transformations

In 1977 Mansouri and Sexl stressed that the Lorentz transformations contain a term which they considered “conventional”, the coefficient of $x$ in the transformation of time. Starting in 1994 I reconsidered the whole matter. I obtained new transformations with an indeterminate term, $e_1$. Omitting the transformations of $x, y, z$, which are identical to the corresponding Lorentz formulae, there remains the transformation of time, given by

$$t = R t_0 + e_1 (x_0 - V t_0)$$

Here $R$ is the usual relativistic velocity dependent square root factor. The Lorentz transformation is recovered as a particular case for $e_1 = -v/c^2 R$. From these formulae one can easily see that the “delay” of a clock in $S$, with respect to the clock in $S_0$ which is passing by, in general depends not only on time, but also on the point $x$ of $S$ in which the former clock is placed. Only if $e_1 = 0$ such a complication is absent. The only remaining unknown term, $e_1$, defines in $S$ the simultaneity of distant events, or, which is the same, chooses the clock synchronization in $S$. Clearly, then, a denomination appropriate for $e_1$ could be “synchronization parameter”. Most experts of relativity consider $e_1$ essentially a free parameter to be fixed by convention, but this book was written precisely to show that physical phenomena require $e_1 = 0$. Different values of $e_1$ imply different empirically equivalent theories. One can check with explicit calculations (to be seen in the next chapter) that the empirical data are often insensitive to the choice of $e_1$ (experiments by Römer, Bradley, Fizeau, Michelson-Morley, Doppler, etc.). There are infinitely many theories explaining the results of these experiments. They are all based on a privileged frame, the only exception being the TSR. The conclusion of physical equivalence of the theories with different $e_1$ seems to agree with the conventionality idea of clock synchronization. There are however some experimental situations of a different type (Sagnac effect, clock paradox, aberration of starlight, linear accelerations…) determining an unique synchronization for obtaining a rational description of the relevant phenomena. The condition $e_1 = 0$ will be obtained six times, independently. The fact that so many proofs exist is a strong indication of the basic coorrectness in nature of absolute simultaneity. “Inertial transformations” is the name proposed for the case $e_1 = 0$. They imply a complete liberation of time from the merely geometrical role to which it had been forced in the Minkowski space and predict that the velocity of light relative to an inertial system $S$ in general is not isotropic. A corresponding anisotropy should exist for Reichenbach’s parameter.
The new transformations

In 1977 Mansouri and Sexl [7-1] stressed that the Lorentz transformations contain a conventional term, the coefficient of the coordinate $x$ in the transformation of time. Starting in 1994 the present author reconsidered the whole matter and reformulated the transformations of the space and time variables between inertial systems [7-2] under very general assumptions. The “equivalent transformations” were obtained containing an indeterminate term, $e_1$, the coefficient of $x$ in the transformation of time: see Eq.s (7.2) below.

The reasoning leading to the equivalent transformations is as follows. Given the inertial frames $S_0$ and $S$ one can set up Cartesian coordinates and make the following standard assumptions:

(i) Space is homogeneous and isotropic and time homogeneous, at least from the point of view of observers at rest in $S_0$;

(ii) Relative to the isotropic system $S_0$ the velocity of light is “$c$ ” in all directions, so that clocks can be synchronized in $S_0$ with the Einstein method and the one way velocities relative to $S_0$ can be measured;

(iii) The origin of $S$, observed from $S_0$, moves with velocity $v < c$ parallel to the $+x_0$ axis, that is according to the equation $x_0 = v t_0$;

(iv) The axes of $S$ and $S_0$ coincide for $t = t_0 = 0$.

The geometrical configuration is thus the usual one of the Lorentz transformations (Fig. 2).
The assumptions (i) and (ii) are not exposed to objections both from the point of view of the TSR and of any plausible theory based on a privileged system; for the TSR they hold in all inertial systems, in the second case they hold in the privileged system only.

In [51] it was shown that the previous conditions reduce the transformation laws from $S_0$ to $S$ to the form

\[
\begin{align*}
x &= f_1(x_0 - \nu t_0) \\
y &= g_2 y_0 \\
z &= g_2 z_0 \\
t &= e_1 x_0 + e_4 t_0
\end{align*}
\] (7.1)

where the factors $f_1, g_2, e_1, e_4$ can depend on the velocity $\nu$ of $S$ measured in $S_0$. An appropriate name for (7.1) could be “general transformations”.

7.1 The velocity of light. Consider any two points $(P_1, P_2)$ of space and let these be described by the space and time variables $P_1(x_{01}, y_{01}, z_{01}, t_{01})$ and $P_2(x_{02}, y_{02}, z_{02}, t_{02})$.

If one writes the system (7.1) two times, with index 1 and with index 2 and subtracts these four equations orderly from one another, then one gets the system (7.1) for the space and time intervals $\Delta x = x_2 - x_1 \ldots \Delta t = t_2 - t_1$ and $\Delta x_0 = x_{02} - x_{01} \ldots \Delta t_0 = t_{02} - t_{01}$. Now invert the obtained result and get $\Delta x_0 = x_{02} - x_{01} \ldots \Delta t_0 = t_{02} - t_{01}$ as functions of $\Delta x = x_2 - x_1 \ldots \Delta t = t_2 - t_1$. This last system of four equations is important also because one can get the transformation of a spherical wave in $S_0$, starting from the geometrical description of the spherical surface emitted at $\Delta t_0 = 0$ from the point with $\Delta x_0 = \Delta y_0 = \Delta z_0 = 0$, that is of

\[
c \Delta t_0 = \left[ \Delta x_0^2 + \Delta y_0^2 + \Delta z_0^2 \right]^{1/2}
\] (7.2)

The equation transformed of (7.2) can be shown to lead to well defined velocity of light one way $c_1(\theta)$ and two way $c_2(\theta)$, both relative to the moving system $S$. Defining $\gamma$ as

\[
\gamma = \frac{f_1}{g_2} \sqrt{1 - \nu^2 / c^2}
\] (7.3)
and introducing polar coordinates for $\Delta x$, $\Delta y$, $\Delta z$, one gets

$$\frac{1}{c_1(\theta)} = \frac{1}{f_1 \left(1 - \frac{V^2}{c^2}\right)} \left[ \left(e_1 + \frac{e_2}{c} V \right) \cos \theta + \left(\frac{e_4}{c} + e_1 \frac{V}{c}\right) \left[\cos^2 \theta + \gamma^2 \sin^2 \theta\right]^{1/2} \right]$$  (7.4)

where $\theta$ is the polar angle, physically the angle between the light propagation direction and the “absolute” velocity of $S$ (parallel to the $x$-axis). Similarly, the two way velocity of light can be shown to be given by

$$\frac{1}{c_2(\theta)} = \frac{1}{f_1 \left(1 - \frac{V^2}{c^2}\right)} \left(\frac{e_4}{c} + e_1 \frac{V}{c}\right) \left[\cos^2 \theta + \gamma^2 \sin^2 \theta\right]^{1/2}$$  (7.5)

Using these results, we will now separately discuss three great ideas of relativistic physics.

**7.2 Lorentz contraction of moving objects.** $P_1$ and $P_2$ are now any two points of a body completely at rest in the system $S$ (no translation, no rotation). Seen from $S_0$ the body is in a state of uniform translation parallel to the $x$ axis. The positions of $P_1$ and $P_2$ satisfy the equations of motion

$$x_{02}(t_0) = vt_0 + x_{02}(0) ; \quad x_{01}(t_0) = vt_0 + x_{01}(0)$$  (7.6)

$$y_{02}(t_0) = y_{02}(0) ; \quad y_{01}(t_0) = y_{01}(0)$$  (7.7)

(only two coordinates for simplicity). Substituting these coordinates in the two first equations (7.1) and subtracting, we get

$$\begin{cases} x_2 - x_1 = f_1 \left[ x_{02}(0) - x_{01}(0) \right] \\ y_2 - y_1 = g_2 \left[ y_{02}(0) - y_{01}(0) \right] \end{cases}$$  (7.8)

Clearly, the Lorentz contraction can be represented by the conditions
The comparison with (7.8) finally gives
\[ f_1 = \frac{1}{\sqrt{1-V^2/c^2}} \quad ; \quad g_2 = 1 \] (7.10)

Obviously, we can conclude as follows: a set of transformations of the space and time variables of the general type (7.1) leads to the Lorentz contraction if and only if the two conditions (7.10) are satisfied.

7.3 Larmor retardation of moving clocks. Let \( Q \) be a clock at rest in \( S \) marking the time \( t \). The equation of motion of \( Q \) seen from \( S_0 \) is
\[ x_0(t_0) = vt_0 + x_0(0) \] (7.11)
Substituting this in the fourth equation (7.1) gives
\[ t = (e_1V + e_4)t_0 + e_4x_0(0) \] (7.12)

Considering any two times \( t_1 \) and \( t_2 \) marked by the moving clock \( Q \) and the corresponding \( S_0 \) times \( t_{01} \) and \( t_{02} \), one easily gets from (7.12)
\[ t_2 - t_1 = (e_1V + e_4)(t_{02} - t_{01}) \] (7.13)

As we know, the retardation of moving clocks in the case considered is given by
\[ t_2 - t_1 = \sqrt{1-V^2/c^2}(t_{02} - t_{01}) \] (7.14)

The comparison with (7.13) finally gives
\[ e_1V + e_4 = \sqrt{1-V^2/c^2} \] (7.15)
Obviously, we can conclude as follows: a set of transformations of the space and time variables of the general type (7.1) leads to the Larmor retardation if and only if the condition (7.15) is satisfied.

7.4 Invariance of the two way velocity of light. A flash of light propagating forth and back on any segment $AB$ at rest in $S$ does so with a two way velocity

$$c_2(\theta) = c$$

(7.16)

independent of $S$ and of the angle $\theta$ formed by the light propagation direction with the $x$ axis. Now let us see the consequences of these ideas applied to the general transformations. First of all, as we saw, $c_2(\theta)$ as given by (7.5) is independent of $\theta$ if and only if $\gamma = 1$. Given (7.3) this is the same as

$$g_2 = f_1 \sqrt{1 - V^2 / c^2}$$

(7.17)

Once (7.17) is satisfied, one has from (7.5)

$$\frac{1}{c_2(\theta)} = \frac{1}{c f_1 (1 - \beta^2)} (e_i V + e_4)$$

(7.18)

At this point it becomes clear that condition (7.16) is satisfied if and only if

$$g_2 = f_1 \sqrt{1 - V^2 / c^2} ; \quad e_i V + e_4 = f_1 (1 - V^2 / c^2)$$

(7.19)

7.5 Lorentz contraction as a consequence of other phenomena. At this point it becomes rather easy to prove the following theorem: “If the Larmor clock retardation and the invariance of the two way velocity of light hold in nature, then the Lorentz contraction holds necessarily as well.” [51] The proof is easy if we set

$$R = \sqrt{1 - V^2 / c^2}$$

(7.20)

and summarize the previous results as follows:

$$(i) \quad \text{Lorentz contraction} \quad f_1 = R^{-1} \quad ; \quad g_2 = 1$$
Larmor retardation
\[ e_1 V + e_4 = R \]

(iii) \[ c_z(\theta) = c \]
\[ g_2 = f_1 R \quad ; \quad e_1 V + e_4 = f_1 R^2 \]

Our first argument can be formalized as follows

\[ (ii) + (iii) \Rightarrow (i) \]

\[ (ii) + (iii)_2 \Rightarrow R = f_1 R^2 \Rightarrow 1 = f_1 R \Rightarrow f_1 = R^{-1}; \]

\[ 1 = f_1 R + (iii)_1 \Rightarrow g_2 = 1; \]

Thus the Lorentz contraction is obtained as a consequence of \((ii)\) and \((iii)\), that is, of two firmly established facts to which the whole chapter 6 was devoted. This shows that the lack of empirical evidence for the Lorentz contraction is more seeming than real. In fact, all the evidence collected for the Larmor retardation and for the invariance of the two way velocity of light can be considered evidence, albeit indirect, for the Lorentz contraction as well.

7.6 The invariant two way velocity of light as a consequence of other phenomena.

Similarly simple, but physically more interesting is the proof that

\[ (i) + (ii) \Rightarrow (iii) \]

\[ (i) \Rightarrow 1 = f_1 R \Rightarrow g_2 = f_1 R \]

\[ (i) \Rightarrow 1 = f_1 R \Rightarrow R = f_1 R^2 \Rightarrow e_1 V + e_4 = f_1 R^2 \]

Thus the invariance of the two way velocity of light is obtained as a consequence of \((i)\) and \((ii)\). The lack of intuitive understanding of the velocity of light behaviour is so superseded by a strict connection with two well defined physical phenomena, the Lorentz contraction and the Larmor retardation.

7.7 The equivalent transformations. Two points discussed in the previous chapter, as we saw, are based on solid empirical evidence:

(P1) The two way velocity of light is the same in all directions and in all inertial systems: \[ c_z(\theta) = c. \]
Clock retardation takes place with the usual factor $R$ when clocks move with respect to $S_0$. Notice that we have now eliminated ambiguities by saying that $R$ in the formula $\tau = \tau_0 / R$ has to be calculated relatively to $S_0$.

These two conditions were shown [51] to reduce the general transformations of the space and time variables from $S_0$ to $S$ to the form

$$\begin{align*}
  x &= \frac{x_0 - vt_0}{R} \\
  y &= y_0 \\
  z &= z_0 \\
  t &= R\tau_0 + e_1 [x_0 - vt_0]
\end{align*}$$

with $R$ given by (7.20). From (7.1) one can easily see that the “delay” $\tau - \tau_0$ of a clock in $S$, with respect to the clock in $S_0$ which is passing by, in general depends not only on $\tau_0$, but also on the point $x$ of $S$ in which the former clock is placed. Only if $e_1 = 0$ such a complication is absent.

The provisionally free parameter $e_1$ defines in $S$ the simultaneity of distant events, or, which is the same, chooses the clock synchronization method to apply in $S$. Clearly, then, a denomination appropriate for $e_1$ is “synchronization parameter”. The only remaining unknown factor is $e_1$. Most experts of the foundations of the relativistic theories consider $e_1$ essentially a free parameter to be fixed by the convention concerning clock synchronization, but the present paper has been written precisely to show the opposite, namely that physical phenomena require a fixed value of $e_1$, precisely $e_1 = 0$. A theory different from the TSR is clearly needed.

Length contraction of rods moving with respect to $S_0$ by the usual factor $R$ (independently of $e_1$) is also a consequence of (7.20). The velocity of light following from (7.20) was obtained:

$$c_1(\theta) = \frac{c}{1 + \Gamma \cos \theta}$$

with

$$\Gamma = \frac{v}{c} + c e_1 R$$

The transformations (7.20) represent the complete set of theories “equivalent” to the TSR: if $e_1$ is varied, different elements of this set are obtained, which, according to the conventionality thesis of Reichenbach, should be equivalent for the explanation of
experimental results. The Lorentz transformation is recovered as a particular case with 
\[ e_1 = -\frac{v}{c^2} R, \] 
whence \( \Gamma = 0 \) and \( c_1(\theta) = c. \) Different values of \( e_1 \) are obtained from 
different synchronization conventions. In all cases but the TSR such values imply the 
existence of a privileged frame.

The Lorentz transformations of the TSR introduce a certain symmetry between space 
variables and time, forcing the latter to a geometrical role in a four dimensional space. With 
Minkowski’s words: “The views of space and time which I wish to lay before you ... are 
radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere 
shadows, and only a kind of union of the two will preserve an independent reality.” [52]

Different values of \( e_1 \) imply different theories of space and time that are empirically 
equivalent to a very large extent. One can check with explicit calculations that the empirical 
data are very often insensitive to the choice of \( e_1 \) (in experiments by Römer, Bradley, Fizeau, 
Michelson-Morley, Doppler, etc.). We will do this in the following section. Thus there are 
ininitely many theories explaining equally well the results of these experiments. It is 
remarkable that all such theories are based on the existence of a privileged frame, the only 
exception being the TSR.

7.8 The inertial transformations. The conclusion of physical equivalence of the theories 
with different \( e_1 \) would seem to agree with the conventionality idea of clock synchronization. 
There are however some experimental situations of a different type (Sagnac effect, clock 
paradox, aberration of starlight, ...) allowing one to determine the synchronization which has 
to be chosen in order to obtain a rational description of natural phenomena. In future chapters 
the condition \( e_1 = 0 \) will be obtained six times, independently. The fact that so many proofs 
exist is a strong indication of the basic correctness in nature of absolute simultaneity. This 
gives rise to the following transformation of space and time:

\[
\begin{align*}
  x &= \frac{x_0 - vt_0}{R} \\
  y &= y_0 \\
  z &= z_0 \\
  t &= Rt_0
\end{align*}
\]  

(7.24)

As already stressed by Mansouri and Sexl [7-1], such transformations would have been the 
logical consequence of the development along the lines of thought of Lorentz-Larmor-
Poincaré: they are the very relations one would write down if one had to formulate a theory in 
which rods shrink and clocks are slow by the usual factor when moving with respect to the 
ether. That the actual development went along different lines was due to the fact that “local 
time” was introduced at the early stage in considering the covariance of the Maxwell
equations. The one way speed of light predicted by (7.24) can be found by taking \(e_1 = 0\) in (7.23):

\[
c_1(\theta) = \frac{c}{1 + \beta \cos \theta}
\]

(7.25)

“Inertial transformations” is the name proposed for the Eq.s (7.24). They imply a complete liberation of time from the merely geometrical role to which it had been forced in the Minkowski space. Furthermore Eq.s (7.24) predict that the velocity of light relative to an inertial system \(S\) moving with respect to the privileged system \(S_0\) is not isotropic. A corresponding anisotropy is predicted for Reichenbach’s parameter \(\epsilon\).

By studying the multiplication properties of the inertial transformations it has been possible to show that they do not form a group. There are no problems with the existence of the identical and inverse transformations, and also the associative law can be satisfied, but it is not always possible to write a meaningful product of two inertial transformations, due to the presence of two absolute velocities \(v\) and \(v'\) in the transformation. If \(\Omega(v, v')\) denotes the transformation (7.24) it is easy to understand that the product \(\Omega(v, v') \Omega(v'', v''')\) is no inertial transformation if \(v'' \neq v'\).

A property implied by (7.24) is absolute simultaneity: two events taking place in different points of \(S\) but at the same \(t\) are judged to be simultaneous also in \(S'\) (and vice versa). The existence of absolute simultaneity does not imply that time is absolute: on the contrary, the \(v\)-dependent factor in the transformation of time gives rise to time-dilatation phenomena similar to those of TSR. A clock at rest in \(S\) is seen from \(S_0\) to run slower, but a clock at rest in \(S_0\) is seen from \(S\) to run faster so that both observers agree that motion relative to \(S_0\) slows the pace of clocks. The difference with respect to TSR exists because in a meaningful comparison of rates a clock at rest in \(S_0\) must be compared with at least two clocks at rest in different points of \(S\), and the result is therefore dependent on the “convention” adopted for synchronizing the latter clocks.

Absolute length contraction can also be deduced from (7.24): All observers agree that motion relative to \(S_0\) leads to contraction. The discrepancy with the TSR is due again to the different “conventions” concerning clock synchronization: the length of a moving rod can only be obtained by marking the simultaneous positions of its end points, and therefore depends on the very definition of simultaneity of distant events.

We conclude the chapter with a short presentation of the history of the transformations alternative to the Lorentz transformations. Perhaps the first proposal was made by H. Ives [7-6] who deduced new transformations by starting from the relativity principle and from a more general “operative principle” by Bridgman. The fundamental parameter of his theory, \(q\), was the absolute velocity of a clock “measured by the clock itself”. Ives believed that the independence of the two way velocity of the state of motion, found by Michelson and Morley, should hold for the one way velocity as well, with a different
numerical value. It was an unlucky conjecture, given that none of the modern equivalent theories has such a property.

Ives’ transformations were very complicated, but had the virtue of considering that the empirical evidence was far from imposing the Lorentz transformations as unavoidable.

In 1961 the American physicist Tangherlini, lecturing on General Relativity at the University of Naples, wrote new transformations \[7-7\] by correcting the Galilei transformations for example with the assumption that a clock in an inertial system in motion with velocity \(V\) does not mark the proper time \(t\) but a slowed down time \(\tau\) given by

\[
d\tau = \sqrt{1 - V^2 / c^2} \, dt
\]

His “corrected Galilean transformations” are of course the (7.5) written above. Immediately after writing the system (1), however, Tangherlini discarded it stating that it requires a synchronization which “cannot be obtained without using either instantaneous signals or the clock synchronization with respect to a different inertial system, things for which there is no justification.”

The idea that the velocity of light is measurable only on closed circuits was discussed also by Edwards [7-8] who built transformations different from the Lorentz ones with particular arbitrary hypotheses on the values of the one way velocity of light along the coordinate axes. We arrive so at 1969, year in which Winnie [7-9] put forward the thesis of conventionality of simultaneity by proposing some new transformations of the space and time variables in which Reichenbach’s parameter \(\varepsilon\) entered explicitly. These transformations reduced to the Lorentz ones for \(\varepsilon=1/2\) and assumed for \(\varepsilon\) a fixed numerical value in every given inertial frame. In this way, however, was missing an essential ingredient of the equivalent transformations, the dependence on space direction of Reichenbach’s parameter \(\varepsilon\) and of the velocity of light. Even if in disagreement with the conventionality of simultaneity thesis, depends strongly on Winnie’s results the family of theories formulated by Vargas and Torr [7-10] which constitute a (not very useful) “covering theory” of relativistic physics and the theory by Ungar [7-11].

In 1977 the important papers by Mansouri and Sexl [7-1] were published. Their declared task was the construction of a test theory of the TSR. For the first time it was explicitly stated that a theory based on absolute simultaneity could be empirically equivalent to the TSR. With this the authors believed to have also established that special relativity is empirically equivalent to an ether theory containing the Lorentz contraction and the retardation of moving clocks. These conclusions were based on the comparison of different synchronization methods and on the study of some empirical situations, like the Römer experiment, the transverse Doppler effect, the Michelson-Morley experiment. The physical arguments of Mansouri and Sexl were certainly of great interest, but the mathematical treatment of the various situations was often not so satisfactory, as based on truncated power series developments.
In 1979 Marinov [7-12] correctly observed that the Michelson-Morley experiment only showed that the bidirectional (i.e., two way) velocity of light is isotropical relative to any inertial system, but gives no information whether the unidirectional (i.e., one way) light velocities are also isotropic.

He assumed that light clocks with equal “arms” at rest in the same inertial system have the same rate, independent of the orientation of their arms. This is clearly the same as the invariance of the two-way velocity of light. He had also a second assumption that was practically equivalent to Lorentz contraction. From these he deduced (in vector form) transformations of space and time equal to the Tangherlini transformations. He wrongly concluded that the new transformations form a group. Concludending pessimistically he wrote that the Lorentz transformations were “much more convenient and productive” than those he had just found.

In the same year Chang [7-13] considered the covariance of Maxwell equations under “Tangherlini transformations”. He predicted the existence of observable differences with respect to the relativistic theory. According to his conclusions near a stationary electric charge in a laboratory on the Earth there should be a magnetic field. His treatment, however, seems influenced by an acritical transport in a theory of notions valid in the TSR (invariance of the line element, tensorial formulation of field theory). Chang also reached the correct conclusion that a theory based on absolute simultaneity is compatible with the existence of superluminal velocities.

In a set of papers starting in 1980 Rembielinski [7-14] wrote the equation for the velocity of light relative to a moving system and obtained essentially our eq. (4). He quoted Tangherlini for having shown that a measurement of the speed of light along any closed trajectory must give $c$ as average value. He stated that Chang’s generalized transformations form a subclass of the nonlinear transformations of the Lorentz group. Such a relativistic covariance in modified form leads to the existence of a privileged reference frame. The works by Rembielinski are of great interest even if difficult to read due to the heavy mathematical formalism.

In this way we arrive at 1986 and Petry’s research [7-15]. His equations (26) were practically coincident with the Tangherlini transformations, which he however called “the Marinov transformations”.

In 1992 an amusing proof of the inertial transformations (in the form inverse of (7.5)) was given by A. Agathangelidis [7-16] using only the normally accepted results of the experiments by Michelson-Morley [7-17], Kennedy-Thorndike [7-18], and Ives-Stilwell [7-19].

Finally, the inertial transformations were called “synchronized transformations” in the excellent 2005 book by de Abreu and Guerra [7-20] which makes good reading for anybody interested in the main lines of thought of the new physics.
[7-16] See the book [RP].
[7-20] See the book [DG].
Chapter 8

Synchronization independent phenomena

Here we are interested in measurements predicted correctly by all the equivalent transformations (ET). In other words the theoretical predictions for the measured physical quantities do not depend on $e_1$. If all physical situations were like those presented here, all theories of a large set would be completely equivalent and the Reichenbach-Jammer conjecture would have been proved correct.

One can start from a generalized Michelson-Morley experiment. From an unique incoming beam of light a semitransparent mirror $P$ generates two coherent beams, which follow different paths until they meet and interfere. The laboratory absolute velocity must be kept into account, as usual with the ET. The calculations turn out to be simple and elegant. All the direction dependent terms cancel with one another. What is left is a set of isotropic terms constituting, by themselves, the relativistic prediction. Therefore all theories based on the ET lead to the same predictions for interferometric experiments of the Michelson type.

In 1851 Fizeau performed an experiment with light propagating in running water repeated by Michelson and Morley in 1886. Light from a source fell on a half silvered mirror, where it divided. The two parts passed inside two tubes $T_1$ and $T_2$, filled with running water, the first (second) part moving parallel (antiparallel) to the current. The interference between the two parts was found in agreement with Fresnel’s partial drag theory for all ET. Furthermore we conclude that a measurement of the time required by light to describe a closed path will necessarily give the same values as the TSR even if moving fluids are present along the path.

In the International Atomic Time system atomic clocks distributed around the world communicate with one another by means of radio signals. The synchronization signal sent by a transmitting station always reaches the receiving station ‘on time,’ at any hour of the day despite the motion of the Earth. According to some authors the proper functioning of this system demonstrates that light, relative to the Earth surface, has the same speed $c$ in all directions. In fact this may not be so; we show that the proper working of the network says nothing about the one way velocity, as it is consistent with another theory, empirically equivalent to the TSR, in which the one way speed of light has a directional dependence in moving frames.

When the Ives-Stilwell experiment was published only the TSR proved to be in agreement with the detection of the transverse Doppler effect. Consequently, the ether hypothesis was considered obsolete for one more reason and the absolute motion was discarded. Now it is interesting to show that it is possible to explain the Doppler effect in the framework of a theory based on the inertial transformations in which the relativity principle does not apply in the usual way. The problem was faced and it was concluded that the Doppler effect is perfectly well explained also within the new theoretical framework.
8. Synchronization independent phenomena

In the present chapter we discuss several famous experiments sharing the property of being explained equally well by all the equivalent transformations (ETs). In other words the theoretical predictions for the measured physical quantities do not depend on the synchronization parameter $e_1$. We start from a generalized version of the Michelson-Morley experiment [8-1].

A laboratory is at rest in the inertial system $S$ moving with absolute velocity $\vec{v}$ and in it an interferometric experiment is performed with a beam of light split into two parts in point $P$ by a semitransparent mirror (Fig. 3). The first part propagates along $P - A_1 - A_2 ... A_m - Q$, with a reflecting mirror placed at intermediate points, oriented so as to produce the right deviation, the second part along the similar path $P - B_1 - B_2 ... B_n - Q$. Finally the two parts superimpose in $Q$ where they interfere, $Q$ being an arbitrary point of an interference figure [8-2]. On the first path we define the vectors $\vec{a}_i$ (with moduli $\ell_{a_i}$), $i = 1, 2, ... m + 1$, coincident with the rectilinear segments described by light and all oriented from $P$ toward $Q$; on the second path we similarly define the vectors $\vec{b}_j$ (with moduli $\ell_{b_j}$), $j = 1, 2, ... n + 1$.

The interference in $Q$ is determined by the time delay $\Delta T$ between the two rays. In the TSR light propagates in all directions with the same speed $c$ and:

$$\Delta T = T_B - T_A = \frac{L_B - L_A}{c}$$

(8.1)

where

$$L_A = \sum_{i=1}^{m+1} \ell_{a_i} ; \quad L_B = \sum_{j=1}^{n+1} \ell_{b_j}$$

(8.2)

Next we calculate $\Delta T$ from the ETs. The inverse velocity of light relative to $S$ is given by Eq. (7.3) and one has:

$$\Delta T = \sum_{j=1}^{n+1} \frac{\ell_{b_j}}{c_1(\theta_{b_j})} - \sum_{i=1}^{m+1} \frac{\ell_{a_i}}{c_1(\theta_{a_i})}$$

(8.3)

where $\theta_{a_i}$ ($\theta_{b_j}$) is the angle between $\vec{a}_i$ and $\vec{v}$ ($\vec{b}_j$ and $\vec{v}$). By inserting (7.3) in (8.3) one obtains:
Figure 3. A semitransparent mirror $P$ generates two coherent beams of light, which follow different paths until they meet and interfere in $Q$. The big grey arrow represents the laboratory absolute velocity.

\[
\Delta T = \frac{L_B - L_A}{c} + \frac{\Gamma}{c} \left( \sum_{j=1}^{n+1} \ell_{bj} \cos \theta_{bj} - \sum_{i=1}^{m+1} \ell_{ai} \cos \theta_{ai} \right)
\]

\[
= \frac{L_B - L_A}{c} + \frac{\Gamma}{c} \left[ \sum_{j=1}^{n+1} \ell_{bj} - \sum_{i=1}^{m+1} \ell_{ai} \right] \frac{\bar{v}}{c}
\]

(8.4)

The last term vanishes because the two terms within curly brackets separately equal the vector joining $P$ and $Q$. Thus (8.1) and (8.4) are the same. Therefore $\Gamma$ (containing $e_1$) disappears from the result and all theories based on the ETs lead to the same predictions for interferometric experiments of the Michelson type.

Next we come to the Fizeau experiment [8-3]. A simple method exists to obtain, consistently with the ETs, the velocity of a flash of light propagating in a medium in motion with respect to an inertial system $S$. Consider the triangle ABC of Fig. 4, at rest in the inertial system $S$, with side lengths $\ell_{ab}$, $\ell_{bc}$ and $\ell_{ca}$ and with suitably oriented mirrors in B and C. The time $t_{abc}$ required by a flash of light to propagate on the closed path ABC can be measured with a single clock in A.
The triangle ABC is at rest in the inertial system $S$. Mirrors in B and C force a flash of light emitted in A to propagate on the closed path ABC. Along AB light propagates in a moving medium independently of synchronization:

$$t_{ABC} = \ell_{AB} c_{AB} + \ell_{BC} c_{BC} + \ell_{CA} c_{CA}$$

(8.5)

where $c_{AB}$ is the velocity of light from A to B, and so on. The sides BC and CA are in the vacuum, while we assume that the path AB is inside a medium (index of refraction $n$) in motion from A to B with velocity $u$ relative to $S$.

In the TSR the velocity of light in such a medium is calculable from the composition of velocities:

$$c_{AB}^{\text{TSR}} = \frac{(c / n) + u}{1 + (u / cn)}$$

(8.6)

In Fig. 4 BC is perpendicular and CA antiparallel to the absolute velocity $v$ of $S$. Therefore, using eq. (7.3) for the light velocity in the vacuum we have
\[ t_{ABC} = \frac{\ell_{AB}}{c_{AB}} + \frac{\ell_{BC}}{c} + \frac{\ell_{CA}}{c} (1 - \Gamma) \]  \hspace{1cm} (8.7)

The prediction of the TSR according to (8.6) is instead

\[ t_{ABC} = \ell_{AB} \frac{1 + (u/cn)}{c/n + u} + \frac{\ell_{BC}}{c} + \frac{\ell_{CA}}{c} \]  \hspace{1cm} (8.8)

But \( t_{ABC} \), measurable with a single clock, is independent of synchronization. Therefore (8.7) and (8.8) must be equal. Considering also that \( \ell_{CA} = \ell_{AB} \cos \theta \) it follows

\[ \frac{1}{c_{AB}} = \frac{n + (u/c)}{c + nu} + \frac{\Gamma \cos \theta}{c} \]  \hspace{1cm} (8.9)

This result shares with eq. (7.3) a property that can be written for any two points \( X \) and \( Y \) connected by light propagation in the vacuum or in a medium (whether at rest or in motion) as follows:

\[ \frac{1}{c_{XY}} = \frac{1}{c_{XY}^{\text{TSR}}} + \frac{\Gamma \cos \theta}{c} \]  \hspace{1cm} (8.10)

where \( \Gamma \), given by eq. (7.4), is independent of the medium. An equivalent expression for the propagation time \( t_{XY} \) over the distance \( \ell_{XY} \) is

\[ t_{XY} = \frac{\ell_{XY}}{c_{XY}} = \frac{\ell_{XY}}{c_{XY}^{\text{TSR}}} + \frac{\Gamma \ell_{XY} \cdot \vec{v}}{c} \]  \hspace{1cm} (8.11)

where \( \ell_{XY} \) is the vector of length \( \ell_{XY} \) oriented from \( X \) to \( Y \).

In 1851 Fizeau [8-3] performed an interferometric experiment with light propagating in running water. His experiment was repeated by Michelson and Morley in 1886 [8-4]. Light from a source at \( \Sigma \) (Fig. 3) fell on a half silvered mirror \( P \), where it divided; one part following the path \( PA_1A_2A_3A_4PD \) and the other the path \( PA_4A_3A_2A_1PD \). The two parts passed inside two tubes \( T_1 \) and \( T_2 \) filled with running water, the first part (second part) moving parallel (antiparallel) to the water current. The interference between the two parts, observed in \( D \), was found to be in agreement with Fresnel’s partial drag theory. Actually we will now discuss a generalized Fizeau experiment [8-5].

Consider a laboratory in an inertial reference frame \( S \) moving with absolute velocity \( \vec{v} \). A light ray propagates along the closed broken path
where suitably oriented reflecting mirrors are placed in the intermediate points (see Fig. 4). The vectors $\vec{\ell}_i$ are defined having moduli $\ell_i$ ($i = 1, 2, \ldots, m + 1$) coinciding with the rectilinear segments described by light and oriented in the propagation direction. We are interested in the total propagation time on the path, $T$. The prediction of the TSR is easy to get, since light moves along $\vec{\ell}_i$ with the velocity

$$c_{A_i \rightarrow A_{i+1}}^{TSR} = c \frac{1 + n_i (u_i / c)}{n_i + (u_i / c)}$$

where $i = 1, 2, \ldots, m + 1$, $A_0$ and $A_{m+1}$ coincide with $P$, $n_i$ is the refraction index of the medium present on the $i$-th segment and $u_i$ its fluid velocity relative to $S$.

One has:

$$T = \sum_{i=1}^{m+1} \frac{\ell_i}{c_{A_i \rightarrow A_{i+1}}^{TSR}}$$

(8.12)

We calculate next the same quantity $T$ by starting from the ETs, according to which we have, applying (8.10) to the segments of the path of Fig. 6:

$$t_{A_i \rightarrow A_{i+1}} = \frac{\ell_i}{c_{A_i \rightarrow A_{i+1}}} = \frac{\ell_i}{c_{A_i \rightarrow A_{i+1}}^{TSR}} + \frac{\Gamma_i \vec{\Omega}_i}{c \nu}$$

(8.13)

From (8.13) it follows:
Notice that \( P{\rightarrow}A_1{\rightarrow}A_2{\ldots}A_m{\rightarrow}P \) is a closed line. Therefore the last term in (8.14) vanishes and (8.14) coincides with (8.12). Therefore \( \Gamma \) (containing \( \epsilon_1 \)) disappears from the result and all theories based on the ETs lead to the same predictions for interferometric experiments of the Fizeau type. We conclude that the ETs predict that a measurement of the time required by light to describe a closed path will necessarily give the same value as predicted by the TSR even if moving fluids are present along the path.

When the Ives-Stilwell experiment was published [8-6] only the TSR proved to be in agreement with the experimental detection of the transverse Doppler effect. Consequently, the ether hypothesis was considered obsolete for one more reason and the absolute motion was discarded. Now it is interesting to study whether it is possible to explain the Doppler effect in the framework of a theory based on the inertial transformations in which the relativity principle does not apply in the usual way. The problem was faced by Puccini and Selleri [8-7] who concluded that the Doppler effect, as well as the results found by Ives and Stilwell, are perfectly well explained also within the new theoretical framework.
We consider the propagation of a light corpuscle \( P \) (a small light pulse propagating in the ray direction) in the privileged frame \( S_0 \), relative to which the velocity of light is assumed to be the same in all directions. We describe \( P \) with coordinates \( x_0 \) and \( y_0 \) satisfying, at time \( t_0 \):

\[
x_0 = c \cos \theta_0 t_0 \quad ; \quad y_0 = c \sin \theta_0 t_0 \tag{8.15}
\]

Our first task is to use the ITs to determine the velocity and the direction of motion of \( P \) in \( S \) in terms of the \( S_0 \) quantities \( c \) and \( \theta_0 \), which are considered given. We consider the inverse inertial transformations from a moving inertial system \( S \) to the privileged one \( S_0 \):

\[
x_0 = R x + \frac{1}{R} v t \quad ; \quad y_0 = y \quad ; \quad t_0 = \frac{1}{R} t \tag{8.16}
\]

Substituting (8.16) into (8.15) we find

\[
x = \frac{1}{R^2} c (\cos \theta_0 - \beta) t \quad ; \quad y = \frac{1}{R} c \sin \theta_0 t \tag{8.17}
\]

Relative to \( S \), which superimposes to \( S_0 \) at time \( t_0 = t = 0 \), the pulse is assumed to have velocity \( c_1 \) and propagation angle \( \theta \), that is, to satisfy equations similar to (1) but with space-time variables \( x, y, t \) and parameters \( c_1 \) and \( \theta \).
Coming to the Doppler effect proper, let us consider a plane electromagnetic
wave (in empty space) described in the privileged frame \( S_0 \) as:

\[
\psi(\vec{r}_0, t_0) = \psi_0 \exp \left\{ i \omega_0 \left( t_0 - \frac{\hat{n}_0 \cdot \vec{r}_0}{c} \right) \right\}
\]  

(8.18)

where \( \psi_0 \) is a constant amplitude, \( \omega_0 \) the angular frequency, \( \hat{n}_0 \) the unit vector normal to the wave fronts and \( c \) the one-way velocity of light. It should be noted that, in the privileged system, \( \hat{n}_0 \) gives the propagation direction of the wave.

In order to study the Doppler effect in a frame \( S \) in motion with a velocity \( \vec{v} \) relative to \( S_0 \), we must substitute \( t_0 \) and \( \vec{r}_0 \) in the phase \( \phi \) of the plane wave (8.18) by the expressions (8.16) of the inertial transformations:

\[
\phi = \omega_0 \left\{ \frac{1}{R} \left( 1 - \frac{\vec{v}}{c} \hat{n}_{0x} \right) t - \frac{1}{c} \left[ n_{0x} Rx + n_{0y} y \right] \right\}
\]  

(8.19)

This can be written

\[
\phi = \omega_0 \left\{ \frac{1}{R} \left( 1 - \frac{\hat{n}_0 \cdot \vec{v}}{c} \right) t - \frac{1}{c} \left[ \hat{n}_0 + \frac{R-1}{v^2} (\hat{n}_0 \cdot \vec{v}) \vec{v} \right] \cdot \vec{r} \right\}
\]  

(8.20)

From (8.20) it is obvious that the angular frequency relative to the \( S \) system is

\[
\omega = \omega_0 \left( 1 - \frac{\hat{n}_0 \cdot \vec{v}}{c} \right)
\]  

(8.21)

which is identical to the relativistic prediction. Therefore the Doppler effect from \( S_0 \) to \( S \) is explained exactly by the ITs, as well as with the TSR.

No need to investigate the Doppler effect with the transformations connecting two moving systems (\( S \) and \( S' \), say). Once \( \omega_0 \) is given, the frequency relative to \( S \) predicted by the inertial transformations from \( S_0 \) to \( S \) is the same as that predicted by the TSR, for all possible \( S \). A transformation from \( S \) to \( S' \) must give the right change of frequency if only the theory is consistent.
Let us come to the discussion of the International Atomic Time system. Atomic clocks distributed around the world communicate with one another by means of radio signals. The synchronization signal sent by a transmitting station always reaches the receiving station 'on time,' at any hour of the day despite the motion of the Earth. In Sexl and Schmidt's opinion [8-8] the proper functioning of this system demonstrates that light, relative to the Earth surface, has the same speed $c$ in all directions. In fact this may not be so; we will now show that the proper working of the network says nothing about the one way velocity, as it is consistent with another theory, empirically (almost) equivalent to the TSR, in which the one way speed of light has a directional dependence in moving frames.
The 1967 General Conference of Weights and Measures redefined the second as the duration of 9,192,631,770 cycles of the radiation absorbed in the transition between two hyperfine levels of Caesium-133 atom. The International Atomic Time (Temps Atomique Internationale = TAI) is the reference time based on the new definition of the second. For establishing TAI the readings of 250 atomic clocks in 45 institutes of the world are systematically compared.

Two stations A and B at some time of S occupy the positions A₁ and B₁; some hours later they have shifted to the new positions A₂ and B₂. We show that the theory of the ITs [8-9] explains the empirical observations just as well as the TSR. According to both theories if a radio signal sent from A at local time tᵦ₁ arrives in B at local time tᵦ₂, the time difference tᵦ₂ − tᵦ₁ (measured on the Earth) is the same from A₁ to B₁ as from A₂ to B₂. These are two generic positions of the stations A and B. The proper working of the TAI, therefore, cannot discriminate the approach based on the ITs from the TSR where the velocity of light relative to the Earth surface is c in all directions.

Consider a rotating circular platform with centre at rest in S. Seen from S, the platform remains circular, in spite of its rotation. In fact we can imagine a pen fixed in a point of the platform border drawing on the ground a closed line γ. This line is completely at rest in S and its shape cannot depend on the chosen clock synchronization. Since in the TSR γ is a circle, it must be seen as a circle in all equivalent theories. Seen from a point vertically above the platform centre, γ overlaps exactly with the border of the rotating platform which, therefore, is also circular. We assume the Earth to be circular.

Velocities will instead depend on clock synchronization. In S the rotation velocity generally is not observed as uniform. Let us see why. Let u₁ and u₂ be the velocities of the Earth surface relative to S on the positions 1 and 2 of Fig. 7. We set, at rest in S near position 1, a suitably oriented segment P–Q with unit length; when the station A passes close to P at velocity u₁, P itself sends out a light signal (at velocity c₁). In Q we measure the time lag between the arrivals of the signal and of the station A. This delay is the difference between the propagation times: 1/u₁ − 1/c₁. This measurement is made in S with one clock only, so the result must be the same that would be obtained according to the TSR, theory in which rotation velocities are isotropic in S. The same operation can be carried out in position 2. Therefore:

\[
\frac{1}{u_1} - \frac{1}{c_1} = \frac{1}{u} - \frac{1}{c} \quad \text{and} \quad \frac{1}{u_2} - \frac{1}{c_2} = \frac{1}{u} - \frac{1}{c} \quad (8.22)
\]

where \( u = \omega r \), \( \omega \) and \( r \) being angular velocity and radius of the Earth in the TSR. Clearly Eq.s (8.22) imply the existence of a first dynamical invariant of the rotating Earth:
According to the theory of ITs the two synchronization signals of Fig. 7 propagate, in $S$, respectively, at:

$$c_1 u_1 = c_2 u_2 \quad (8.23)$$

Replacing (8.24) in (8.22) one easily gets

$$u_1 = \frac{uc}{c + u\beta\cos\theta_1} \quad \text{and} \quad u_2 = \frac{uc}{c + u\beta\cos\theta_2} \quad (8.25)$$

Thus the platform rotation velocities in the points $A_1$ and $A_2$ are different. If the segments $A_1B_1$ and $A_2B_2$ of Fig. 5, judged to be of equal length in the TSR, had equal length also in our approach two observers at rest in $S$ near the points $A_1$ and $A_2$ would see different numbers of such segments pass by in the unit time. This is absurd, as it would imply an accumulation of matter on one side; we write $A_1B_1 = \ell_1$ and $A_2B_2 = \ell_2$, where, in general, $\ell_1 \neq \ell_2$.

The station $B$ on the border of the rotating platform passes near a clock $\Gamma$ at rest in $S$ when this clock marks the time $t$. An observer checks on $\Gamma$ the time $t + \Delta t$ at which also the station $A$ later passes near $\Gamma$. The delay $\Delta t$, measured with a single clock, cannot depend on synchronization. Therefore it is the same in all equivalent theories and has the value predicted by the TSR in which the platform rotation is uniform. Thus $\Delta t$ has a value independent on the position of $\Gamma$ in $S$ near the platform border: in this sense rotation is uniform in all the ETs.

In the positions 1 and 2 of Fig. 5 the time interval $\Delta t$ starts when the station $B$ passes near $\Gamma$ and ends when the station $A$ arrives near $\Gamma$. The distances $\ell_1$ and $\ell_2$ are traversed by $A$ with velocities $u_1$ and $u_2$, respectively (all measured in $S$). Then $\Delta t = \ell_1 / u_1$ and $\Delta t = \ell_2 / u_2$. This gives a second dynamical invariant:

$$\frac{\ell_1}{u_1} = \frac{\ell_2}{u_2} \quad (8.26)$$

A third invariant can be shown to exist, if $u_1$ and $u_2$ are the rotation velocities of the stations (measured in $S$). A short calculation leads to finds

$$\frac{R_1}{u_1} = \frac{R_2}{u_2} \quad (8.27)$$
Three dynamical invariants of the rotating Earth. By multiplying them together one

\[
\frac{R_1 \ell_1}{u_1} \frac{c_1 u_1}{c_1 - u_1} = \frac{R_2 \ell_2}{u_2} \frac{c_2 u_2}{c_2 - u_2} \tag{8.28}
\]
a result which will be very useful. All the following calculations are performed from the point of view of the inertial system \(S\) relative to which the Earth rotates without translating. We want to calculate the times taken by the two synchronization signals to travel from station \(A\) to station \(B\) in the positions 1 and 2 of Fig. 7. Therefore we are interested in the timing of four event as shown by clocks at rest in \(S\) near the points where the events take place:

- \(t_{A1}\): the first radio signal leaves \(A\);
- \(t_{B1}\): the first radio signal arrives in \(B\);
- \(t_{A2}\): the second radio signal leaves \(A\);
- \(t_{B2}\): the second radio signal arrives in \(B\).

For \(t_{B1}\) we can write

\[
t_{B1} = t_{A1} + \frac{\ell_1}{c_1 - u_1} \tag{8.29}
\]
where the last term is the pulse propagation time, justified as follows.

The observer in \(S\) sees the radio signal and the station \(B\) moving in well defined ways, that is according to the equations \(\xi = c_1 \left(t - t_{A1}\right)\) and \(\xi = \ell_1 + u_1 \left(t - t_{A1}\right)\), respectively, if \(\xi\) is a coordinate on the \(AB\) line with origin in \(A\). Clearly the equal position condition (arrival of the signal in \(B\)) will be obtained after a time interval \(t - t_{A1}\) given by \(\ell_1 / \left(c_1 - u_1\right)\).

Let us come to the second position. For \(t_{A2}\) one can write

\[
t_{A2} = t_{A1} + \frac{\ell_1}{u_1} + T_{12} \tag{8.30}
\]
where \(\ell_1 / u_1\) is the station \(A\) propagation time from \(A_1\) to \(B_1\) and \(T_{12}\) is the station \(A\) propagation time from \(B_1\) to \(A_2\) (see Fig. 8).

Finally, for \(t_{B2}\) one can write

\[
t_{B2} = t_{A1} + T_{12} + \frac{\ell_2}{u_2} + \frac{\ell_2}{c_2 - u_2} \tag{8.31}
\]
where $T_{12}$ is the station $B$ propagation time from $B_1$ to $A_2$ ($T_{12}$ is obviously the same for the two stations) and $\ell_2/\ell u_2$ is the station $B$ propagation time from $A_2$ to $B_2$. The last term in (8.31) is the pulse propagation time.

Seen from the paths followed by the stations $A$ and $B$, due to the rotation of Earth, are not the same (Fig. 8); the symmetry is broken by the presence of a privileged direction, that of the absolute $v$ of translation with which the rotation velocity composes. This point is crucial. The segment $B_1 - A_2$ is common to the two paths and does not introduce any difference. The segment $A_1 - B_1$ is travelled only by the station $A$ at a faster absolute velocity, compared to the segment $A_2 - B_2$ which is travelled only by the station $B$. The times marked by the two clocks, in these two segments, are different: the clock in $A$, going faster, develops a delay compared with the one in $B$.

How will these times be perceived on the Earth? To answer we perform ITS from $s$ to the inertial systems $S'_1$ and $S'_2$ in which the stations $A$ and $B$ can be considered at rest during the short time in which $A$, seen from $s$, moves either from $A_1$ to $B_1$ or from $A_2$ to $B_2$. According to the golden rule of the time transformation applied separately to the (small) regions 1 and 2 of Fig. 5 the $s$ times will be registered by the stations on the Earth slowed down by $R_1/R$ (if in region 1) and by $R_2/R$ (if in region 2). For $t'_{B1}$ we obtain

$$t'_{B1} = t'_{A1} + \tau' + \frac{R_1}{R c_1 - u_1} \ell_1$$

(8.32)
While station $A$ moves from $A_1$ to $A_2$, station $B$ moves from $B_1$ to $B_2$. The path $B_1 A_2$ is common to the two stations and differences can only arise from $A_1 B_1$ and $A_2 B_2$.

where $\tau'$ is the conventional time actually added to the time marked by the clock of the $B$ station in order to achieve a particular synchronization, e.g. the one ensuring that the Earth based measurement of the velocity of a radio pulse from $A$ to $B$ is $c$.

Let us come to the second position. For $t'_{A2}$ we can write

$$t'_{A2} = t'_{A1} + \frac{R_1}{R} \frac{\ell_1}{u_1} + T'_{12}$$

(8.33)

because the station $A$ propagation from $A_1$ to $B_1$ takes place in region 1. In (8.33) $T'_{12}$ is the station $A$ propagation time from $B_1$ to $A_2$ as measured on the Earth. Finally, for $t'_{B2}$ we have

$$t'_{B2} = t'_{A1} + \tau' + T'_{12} + \frac{R_2}{R} \left[ \frac{\ell_2}{u_2} + \frac{\ell_2}{c^2 - u_2} \right]$$

(8.34)
where $T'_{12}$ is the station $B$ propagation time from $B_1$ to $A_2$ as measured on the Earth. Again, it is the same as the station $A$ propagation time from $B_1$ to $A_2$. From (8.32)-(8.34) we obtain the time differences

$$t'_{B_1} - t'_{A_1} = \tau' + \frac{R_1}{R} \frac{\ell_1}{c_1 - u_1}$$

(8.35)

and

$$t'_{B_2} - t'_{A_2} = \tau' + \frac{R_2}{R} \left(\frac{\ell_2}{u_2} + \frac{\ell_2}{c_2 - u_2}\right) - \frac{R_1}{R} \frac{\ell_1}{u_1}$$

(8.36)

The latter equation is the same as

$$t'_{B_2} - t'_{A_2} = \tau' + \frac{R_2}{R} \frac{\ell_2}{u_2} \frac{u_2c_2}{c_2 - u_2} - \frac{R_1}{R} \frac{\ell_1}{u_1}$$

(8.37)

Due to the product of the three invariants the second term in the right hand side can be written with the index everywhere changed from 2 to 1. It is now very easy to check that

$$t'_{B_2} - t'_{A_2} = t'_{B_1} - t'_{A_1}$$

(8.38)

Therefore the two stations will not detect any desynchronization between the clocks. On the surface of the Earth it so happens that the delay due to the different rates of the two clocks compensates exactly the difference in the times taken by the two synchronization signals to travel from $A$ to $B$.

We applied the ITs to small regions of the rotating Earth. By doing this we used a principle: a small region of an accelerated frame is physically equivalent to the ‘comoving’ inertial frame having the same instantaneous velocity. The same principle is applied by people using the relativistic idea of the invariance of the velocity of the electromagnetic radiation. With the Lorentz transformations $c$ is isotropic everywhere and the clocks of two different stations must remain synchronous, as observed. With the inertial transformations there are two phenomena producing a desynchronization: the first one is the anisotropy of the velocity of light, the other one the variable absolute velocity of the clocks generating differences in their pace. The two effects are equal and opposite and cancel, so that the clocks always appear to be synchronous when a signal connects them. The proper functioning of the world time system does not say anything about the one way speed of light and cannot establish which theory is “true”. Thus the theory of the inertial transformations is, in this respect, completely equivalent to the TSR.


Chapter 9

The Sagnac effect: $e_1 = 0$

It is remarkable that almost a century after its discovery no theoretical justification of the Sagnac effect has been obtained from the two relativistic theories. Hasselbach and Nicklaus listed about twenty different explanations of the Sagnac effect, but this plurality is a clear sign of weakness. When a physical phenomenon has a rational interpretation there is no space for disagreements. Sagnac published two papers (in French) with titles like “The existence of the luminiferous ether demonstrated.” One would have expected a strong reaction, but the first comments appeared after 8 years in a paper by Langevin, the preeminent French physicist of the time. Langevin argued that Sagnac’s was a first order experiment, on which all theories had to agree quantitatively, given that the experimental precision did not allow one to detect second order effects: therefore it could not produce evidence for or against any theory. Then he showed that Galilean kinematics explained the empirical observations! In fact his approach was only slightly veiled in relativistic form by some words and symbols, but was essentially Galilean.

The impression that Langevin, beyond printed words, could not be satisfied with his explanation is reinforced by his second article (1937) in which two relativistic treatments are presented. The first is still that of 1921, this time deduced from the strange idea that the time to be adopted everywhere on the disk is that of the rotational center (motionless in the laboratory). The second one is to define “time” in such a way as to enforce a velocity of light constant and equal to $c$. The 1963 review paper by Post seems to agree with the idea that two relativistic proofs of the Sagnac effect are better than one. The first proof (in the main text) uses arbitrarily the laboratory to platform transformation of time $t' = t + \boldsymbol{v} \cdot \hat{r} / c^2$, but it hastens to make the second term disappear with the (arbitrary) choice of $\hat{v} \cdot \hat{r} = 0$. The tendency by Langevin and Post to cancel the $x$ in the transformation of time shows once more the great difficulty in explaining the physics of the rotating platform with the relativistic theory. Somehow it anticipates the approach based on the inertial transformations, the only to explain the Sagnac effect. Also the space devoted by the Landau and Lifshitz book to the question of clock synchronization on the rotating platform is interesting, especially because the authors do not spend a single critical word for the relativistic theory while they advance in the intricacies of two clocks in fixed positions having different synchronizations relative to one another, depending on the path followed for reaching the respective positions on the platform. Many other papers have been published but the situation did not change.
9. **The Sagnac effect: $e_1 = 0$**

It is remarkable that almost a century after the 1913 discovery of the Sagnac effect no theoretical justification based on the two relativistic theories has been found. Hasselbach and Nicklaus, describing their own experiment [9-1], list about twenty different explanations of the Sagnac effect and comment: “This great variety (if not disparity) in the derivation of the Sagnac phase shift constitutes one of the several controversies ... that have been surrounding the Sagnac effect since the earliest days of studying interferences in rotating frames of reference.” In this section the reasons for this strange resistance of the relativistic theories at explaining the Sagnac effect are pointed out. A fully satisfactory explanation is shown to be available, only within a theory based on absolute simultaneity ($e_1 = 0$).

In the Sagnac 1913 experiment a platform was made to rotate uniformly around a vertical axis at a rate of 1-2 full rotations per second. In an interferometer mounted on the platform, two interfering light beams, reflected by four mirrors, propagated in opposite directions along a closed horizontal circuit defining a certain area $A$. The rotating system included also the luminous source and a photographic plate recording the interference fringes. On the pictures obtained during a clockwise and a counterclockwise rotation with the same frequency, Sagnac observed the interference fringes in different positions and measured the displacement $\Delta z$ by overlapping the two figures.

This $\Delta z$ is strictly tied to the relative time delay with which the two light beams reach the detector. Sagnac observed a shift of the interference fringes every time the rotation was modified. Considering his experiment conceptually similar to the Michelson-Morley one, he informed the scientific community with two papers (in French) bearing the titles “The existence of the luminiferous ether demonstrated by means of the effect of a relative ether wind in an uniformly rotating interferometer” [9-2] and “On the proof of reality of the luminiferous ether with the experiment of the rotating interferometer” [9-3]

The experiment was repeated many times in different ways, with the full confirmation of the Sagnac results. Famous is the 1925 repetition by Michelson and Gale [9-4] for the very large dimensions of the optical interference system (a rectangle about 650m x 360m); in this case the disk was the Earth itself at the latitude concerned. The light propagation times were not the same, as evidenced by the resulting fringe shift. Full
consistency was found with the Sagnac formula [see below] if the angular velocity of the Earth rotation was used.

Figure 9. Simplified Sagnac apparatus. Light from a source $S$ is divided in two parts by the semitransparent mirror $A$. The first part follows the path $ABCDAO$ concordant with rotation, the second part follows $ADCBAO$ discordant from rotation. Interference fringes observed in $O$.

Can light propagate with the usual velocity $c$ relatively to the rotating platform? The question was directly faced in the 1942 experiment by Dufour et Prunier [9-5], in which the mirrors defining the paths of the interfering light beams were partly fixed in the laboratory (directly above the disk) and partly in the spinning disk. The fringe shifts were the same as in a repetition of the test with all mirrors fixed on the disk, confirming that the light does not adapt to the movement of the disk, and that it is physically connected with some other reference system, in all probability inertial.
Surprisingly theoreticians were little interested in the Sagnac effect, as if it did not pose a conceptual challenge. For a presentation of Einstein’s ideas about the rotating disk see a paper by Stachel [9-6]. As stated before, the first discussion by Langevin came only 7-8 years later [9-7] and was as much formally selfassured as substantially weak. One of the opening statements is this: “I will show how the theory of general relativity explains the results of Sagnac’s experiment in a quantitative way.” Langevin argues that Sagnac’s is a first order experiment, on which all theories (relativistic or prerelativistic) must agree qualitatively and quantitatively, given that the experimental precision does not allow one to detect second order effects: therefore it cannot produce evidence for or against any theory. Then he goes on to show that an application of Galilean kinematics explains the empirical observations! In fact his approach is only slightly veiled in relativistic form by some words and symbols, but is essentially 100% Galilean.

The impression that Langevin, beyond printed words, could not be satisfied with his explanation is reinforced by his second article of 1937 [9-8] in which two relativistic treatments are presented. The first one is still that of 1921, this time deduced from the strange idea that the time to be adopted everywhere on the disk is that of the rotational centre (motionless in the laboratory). The second one is to define “time” in such a way as to enforce a velocity of light constant and equal to $c$, falling so flatly in the problem of the discontinuity for a tour around the disk that we will discuss later.

The 1963 review paper by Post [9-9] seems to agree with the idea that two relativistic proofs of the Sagnac effect are better than one. The first proof (in the main text) uses arbitrarily the laboratory to platform transformation of time $t' = tR$ where $R$ is the usual square root factor of relativity, here written with the rotational velocity. The second proof (in an appendix) starts from the correct Lorentz transformation $t' = (t + \vec{v} \cdot \vec{r}/c^2)/R$, but it hastens to make the second term disappear with the (arbitrary) choice of $\vec{r}$ perpendicular to $\vec{v}$.

The tendency by Langevin and Post to cancel the spatial variable $x$ in the transformation of time shows once more the great difficulty in explaining the physics of the rotating platform with the relativistic theory. Somehow it anticipates the approach based on the inertial transformations, the only ones which can explain the Sagnac effect. Also the space devoted by the Landau and Lifshitz book [9-10] to the question of clock synchronization on the rotating platform is interesting, especially because
the two Russian physicists do not spend a word in criticizing the relativistic theory while they advance in the intricacies of two clocks in fixed positions having different synchronizations relative to one another, depending on the path followed for reaching the respective positions on the platform.

Vetharaniam and Stedman [9-11] developed a theoretical model supposed to describe Lorentz invariance locally, that is for an infinitesimal time interval and in an infinitesimal region of space in an accelerated reference frame. They claim that the model can be tested by using a precision ring laser to bound parameters of the theory. I can only remark that the validity of the local Lorentz invariance is immediately in contradiction with the very existence of the Sagnac effect. In fact, Lorentz invariance implies equal local velocities of light along opposite directions and thus - given the circular symmetry of the problem - also equal global velocities of light along opposite directions. After full tours in opposite directions along the rim of the rotating disk, the two pulses would hit the target at the same time of the disk and the Sagnac effect would disappear.

A very interesting modified Sagnac experiment has been carried out recently by Ruyong Wang and collaborators. [9-12]. The instrument was designed to decide whether the travel time difference only appears in rotational motion, or if it also appears in rectilinear uniform motion. The results were unequivocally in favour of the latter possibility, in full agreement with our present approach to relativistic physics, which attributes in all cases the same local velocity relative to an accelerated reference frame and to the locally comoving inertial frame.

Next we come to the main point: the inertial transformations provide a complete qualitative and quantitative explanation of the Sagnac effect [9-13]. In order to simplify calculations, we consider now a monochromatic light source $\Sigma$, placed on the disk, emitting two pulses of light in opposite directions. The description of light propagation given by the laboratory observers is the following: two light flashes leave $\Sigma$ at time $t_{01}$. The first one propagates on a circumference, in the sense discordant from the platform rotation, and comes back to $\Sigma$ at time $t_{02}$ after circling around the platform. The second flash propagates on the same circumference, in the sense concordant with the platform rotation, and comes back to $\Sigma$ at time $t_{03}$ after circling around the platform. These laboratory times, all relative to events taking place in a fixed point of the platform very near $C_\Sigma$, are related to the corresponding platform times via
The circular path can be obtained by forcing the light to propagate tangentially to the internal surface of a cylindrical mirror. Most textbooks deduce the Sagnac formula (our Eq. (9.6) below) in the laboratory, but say nothing about the description of the phenomenon given by an observer placed on the rotating platform: we will see that the TSR predicts a null effect on the platform, while our approach based on the inertial transformations gives the right answer. For simplicity we will assume that the laboratory is at rest in the privileged frame.

Sagnac effect seen from the laboratory. Light propagating in the rotational direction of the disk must cover a distance larger than the disk circumference length $L_0$ by a quantity $\xi = v(t_{02} - t_{01})$ equaling the shift of $S$ during the time $t_{02} - t_{01}$ taken by light to reach the interference region. Therefore

$$L_0 + \xi = c(t_{02} - t_{01}) ; \quad \xi = v(t_{02} - t_{01}) \quad (9.2)$$

From these equations it is easy to get:

$$t_{02} - t_{01} = \frac{L_0}{c - v} \quad (9.3)$$

Light propagating in the direction opposite to rotation must instead cover a distance smaller than the disk circumference length $L_0$ by a quantity $\eta = v(t_{03} - t_{01})$ equaling the shift of $A$ during the time $t_{03} - t_{01}$ taken by light to reach the interference region. Therefore

$$L_0 - \eta = c(t_{03} - t_{01}) ; \quad \eta = v(t_{03} - t_{01}) \quad (9.4)$$

One now gets

$$t_{03} - t_{01} = \frac{L_0}{c + v} \quad (9.5)$$

$$t_{0i} = t_i \sqrt{1 - v^2 / c^2} \quad (i=1,2,3) \quad (9.1)$$
The time difference \( \Delta t_0 \) between the two propagation times is the parameter fixing the phase difference in the considered interference point. One can easily see that:

\[
\Delta t_0 = t_{03} - t_{02} = (t_{03} - t_{01}) - (t_{02} - t_{01})
\]

From (9.3) and (9.5) it then follows

\[
\Delta t_0 = t_{03} - t_{02} = \frac{2L_0}{c^2} \frac{\nu}{1 - \nu^2 / c^2} = \frac{2L \nu}{c^2 R} \tag{9.6}
\]

Obviously \( L_0 = LR \) is the disk circumference length reduced in the laboratory by the usual relativistic factor, if \( L \) is the rest length of the same disk. On this point it is better to avoid misunderstandings. I repeat: (i) \( L_0 \) is the disk circumference length measured by observers at rest in the laboratory who see the disk in a state of rotation; (ii) \( L \) is the disk circumference length measured by observers at rest on the disk. From these definitions it appears clearly that the equation \( L_0 = LR \) is nothing but the Lorentz contraction of the disk perimeter.

The consistency of (9.6) with experimental data has been checked in many experiments.

Sagnac effect seen from the disk. As a preliminary to the solution of the problem on the disk consider a clock marking the time \( t \) fixed in the origin of the moving inertial system \( S \). Seen from \( S \), it therefore satisfies the equation \( x_0 = \nu t_0 \). Substituting this equation into the equivalent transformations (7.2) we get \( x = 0 \) (the fixed coordinate of the clock in \( S \)) and \( \tau = \tau_0 \). Therefore, all the ETs lead to the same relationship between the times \( t, t_0 \). For time intervals between two events we write

\[
\Delta t = R \Delta t_0 \tag{9.7}
\]

Eq. (9.7) will be assumed to hold also for a clock on the rim of a disk rotating with speed \( \nu \). This is consistent with our general philosophy that every small portion of the circumference of the rotating platform can be considered instantaneously at rest in a moving inertial frame of reference.
locally “tangent” to the disk. Therefore Eq. (7.3) applies for the velocity of light on the disk. Only the cases of light moving parallel and antiparallel to the local absolute velocity must be considered. It follows from (7.3) that the inverse velocity of light for these two cases is respectively given by:

\[
\frac{1}{\tilde{c}(0)} = \frac{1 + \Gamma}{c} \quad ; \quad \frac{1}{\tilde{c}(\pi)} = \frac{1 - \Gamma}{c}
\]  

(9.8)

with \( \Gamma \) given by (7.4). The time difference on the disk is given by

\[
\Delta t = t_1 - t_2 = \frac{L}{\tilde{c}(0)} - \frac{L}{\tilde{c}(\pi)} = \frac{2L\Gamma}{c}
\]

(9.9)

Substituting (9.6) in the right hand side of (9.9) we get

\[
\Delta t = \Delta t_0 R \left[ 1 + \frac{c^2 e_1 R}{v} \right]
\]

(9.10)

where \( R \) is the usual square root factor describing the dilation of time intervals in a moving frame. Now, the result aimed at is (9.6): only the inertial transformations, corresponding to \( e_1 = 0 \) allow us to get (9.6) from (9.10). For all other values of \( e_1 \) one will get wrong results from (9.10). In particular, the TSR with its \( e_1 = -v/c^2R \) predicts \( \Delta t = 0 \).

We have reached the conclusion that of all theories having different values of \( e_1 \) only one (\( e_1 = 0 \)) gives a rational description of the Sagnac effect on the rotating platform. In the case of \( e_1 \neq 0 \) the calculated time difference on the platform disagrees with the prediction (9.6) in the laboratory, prediction which is of course the same for all theories satisfying the equivalent transformations (SRT included), since in the laboratory (assumed to be at rest in the privileged frame) Einstein’s synchronization was used.

The Sagnac effect is also important for understanding the nature of the so called Sagnac correction on the Earth surface. As recounted by Kelly [9-14], in 1980 the CCDS (Comité Consultatif pour la Définition de la Seconde) and in 1990 the CCIR (International Radio Consultative Committee) suggested rules - later universally adopted - for synchronizing
clocks in different points of the globe. Two are the methods used to accomplish this task. The first one is to transport a clock from one site to another and to regulate clocks at rest in the second site with the time reading of the transported clock. The second method is to send an electromagnetic signal informing the second site of the time reading in the first site. The rules of the committee establish that three corrections should be applied before comparing clock readings:

(a) the first correction keeps into account the velocity effect of the theory of special relativity (TSR). It is proportional to $v^2 / 2c^2$, where $v$ is the velocity of the airplane, and corresponds to a slower timing of the transported clock;

(b) the second correction keeps into account the gravitational effect of the theory of general relativity (TGR). It is proportional to $g(\phi)h / c^2$ where $g$ is the total acceleration (gravitational and centrifugal) at sea level at the latitude $\phi$ and $h$ is the height over sea level. It corresponds to a faster timing of the transported clock;

(c) the “Sagnac correction” is assumed proportional to $2A_E \omega / c^2$, where $A_E$ is the equatorial projection of the area enclosed by the path of travel of the clock (or of the electromagnetic signal) and the lines connecting the two clock sites to the centre of the Earth, and $\omega$ is the angular velocity of the Earth.

There are no doubts about nature and need of the first two corrections, but the justification of the third one is unconvincing. I agree completely with Kelly [9-15] when he says that the only possible reason to include (c) is that the eastward velocity of light relative to the Earth is different from the westward.

In fact one can deduce, for a real experiment, the “Sagnac correction” from Eq. (9.1) applied to a geostationary satellite, for which the satellite itself and the Earth surface can be thought to be at rest on the same rotating platform.

Saburi et al. carried out their experiment in 1976, before the CCDS and CCIR deliberations, and made clear that “corrections” were indeed necessary already in the title of their paper [9-16] (“High-Precision Time Comparison via Satellite and Observed Discrepancy of Synchronization”).
They had two atomic clocks, not quite synchronous, one in a first station \( W \) (near Washington, USA) the other one in a second station \( T \) (near Tokyo, Japan) practically on the same parallel of the two cities. The time difference between the two clocks on August 27, 1976 was measured with two different methods:

(i) by sending an airplane carrying a third clock (initially synchronous with the one in \( W \)) from \( W \) to \( T \), via Hawaii (westward);

(ii) by sending an electromagnetic signal, via a geostationary satellite, from \( W \) to \( T \), again westward.

The uncorrected airplane clock found the \( T \) clock 9.42 \( \mu s \) fast with respect to the \( W \) clock. The velocity correction and the gravitational correction together were estimated to be about 0.080 \( \mu s \) (to be subtracted to the time shown by the transported clock). By applying such a correction the \( T-W \) time difference increased to 9.50 \( \mu s \).

The electromagnetic signal carried with itself the time shown by the clock of the transmitting station. Assuming that the signal velocity was \( c \), it was found that the \( T \) clock was 9.11 \( \mu s \) fast with respect to the \( W \) clock. Thus, the discrepancy between the two measurements was about 0.39 \( \mu s \).

Let \( L_{WS} \) and \( L_{ST} \) the Washington-satellite and Tokyo-satellite distances, respectively (see Fig. 10). As most physicists in similar experiments, Saburi and collaborators synchronized clocks by imposing that the velocity of light is \( c \), that is in such a way that \( t_T - t_W = (L_{WS} + L_{ST})/c \), \( t_W \) and \( t_T \) being the times of signal departure from \( W \) and arrival in \( T \) as marked by the respective clocks. In order to ensure that this formula was correct for their clocks they had to apply the so called “Sagnac correction” to the clock of the receiving station. Such a correction is given by \( \Delta t_T = 2 \omega A_E/c^2 \) where \( A_E \) is the area of the quadrangle \( OWSTO \) of Fig. 9.

By adopting such an approach Saburi and collaborators made an error because, as we know, the correct velocity of light relative to the rotating Earth is that given by the inertial transformations, which in the appropriate directions is

\[
c_{WS} = \frac{c}{1 + \beta \cos \alpha_{WS}} \quad ; \quad c_{WS} = \frac{c}{1 + \beta \cos \alpha_{WS}}
\]  

(9.11)
where $\beta = \omega r / c$ ($r$ is the radius of the $W$-$T$ parallel and $\omega$ is the Earth angular velocity), $\alpha_{WS}$ is the angle between the line $WS$ and the local velocity (normal to $OW$), $\alpha_{ST}$ is the angle between the line $ST$ and the normal $OT$ in Fig. 10. Therefore $\alpha_{WS} = \theta_w - \pi/2$ and $\alpha_{ST} = \theta_r - \pi/2$ where $\theta_w$ and $\theta_r$ are the angles $OWS$ and $OTS$ of Fig. 10, respectively.
Using the definition $\beta = \omega r / c$ one can write

$$\Delta T = \omega r (L_{WS} \cos \alpha_{WS} + L_{ST} \cos \alpha_{ST}) / c^2$$

but

$$r L_{WS} \sin \theta_W + r L_{ST} \sin \theta_T = 2 A_E$$

where $A_E$ is the area of the quadrangle $OWSTO$ of Fig. 10. We have thus provided a full physical justification of the Sagnac correction $2A_E \omega / c^2$.

We see that the mystery of the “Sagnac correction” of Earth physics disappears with the inertial transformations.

Eq. (9.10) is not the velocity adopted in this experiment. Having imposed the impossible condition that the velocity of light is $c$ the quoted authors had now to apply the mysterious “Sagnac correction” $\Delta t_T$ on the time of arrival in $T$. Such a correction, from our point of view, is best calculated by replacing $c$ with $c_{WS}$ and $c_{ST}$ as follows

$$\Delta t_T = \frac{L_{WS}}{c_{WS}} - \frac{L_{WS}}{c} + \frac{L_{ST}}{c_{ST}} - \frac{L_{ST}}{c}$$

(9.12)

which is positive as $c > c_{WS}, c_{ST}$.

Our results confirm the qualitative observation of Hayden [9.17]: electromagnetic signals need more time for a full tour around our planet toward east than toward west and this can only mean that relatively to the Earth the velocity of light in the two senses is not the same.
Besides the century old problem of the Sagnac effect the relativistic description of rotating platforms contains another fundamental difficulty, probably related to the former. Once more the difficulty can be eliminated by substituting in inertial systems the Lorentz transformations with the IT based on $e_1 = 0$.

It is well known that no perfectly inertial frame exists because of Earth rotation, of orbital motion around the Sun, of Galactic rotation. All knowledge about inertial systems has been obtained in systems having small but non zero acceleration $a$. For this reason the limit $a \to 0$ taken in the theoretical schemes should be smooth and no discontinuity should arise between systems with small acceleration and inertial systems. This requirement is not satisfied by the TSR. Consider an isotropic inertial reference frame $S_0$ so that the one-way velocity of light in $S_0$ has the usual value $c$ in all directions. In relativity this condition holds for all inertial frames, while in ether theories only the privileged frame satisfies it.

A circular platform (radius $r$, center at rest in $S_0$) rotates uniformly around its axis with angular velocity $\omega$ and peripheral velocity $v = \omega r$. On its rim there is a clock $\Sigma$. The motion modifies the pace of $\Sigma$ and the relationship between the times $t$ of $\Sigma$ and $t_0$ of $S_0$ is taken to be $t_0 = t F$, where $F$ is a function of $v$ and eventually of higher derivatives of position. This obvious consequence of the isotropy of $S_0$, was confirmed by the CERN storage ring measurements on the muon, which gave $F = R^4$.

The clock acts also as light source. Two light flashes leave $\Sigma$ and are forced to move on the rim, by “sliding” on the internal surface of a cylindrical mirror placed at rest on the platform, all around it very near its border.

If $\tilde{c}(0)$ and $\tilde{c}(\pi)$ are the light velocities, relative to the disk, for the flash propagating in the direction of the disk rotation and in the opposite direction, respectively, one obtains with a few elementary steps using the definition of velocity two formulae whose ratio $\rho \equiv \tilde{c}(\pi)/\tilde{c}(0)$ turns out to satisfy

$$\rho = \left(1 + \beta \right) / \left(1 - \beta \right)$$  \hspace{1cm} (R)
Eq. \((R)\) will apply also to the ratio of the instantaneous velocities [thus we do not need a different symbol for the instantaneous velocities].

The result \((R)\) holds with the same numerical value for platforms having different radius, but the same peripheral velocity. A small part \(AB\) of the rim of a platform, having peripheral velocity \(\nu\) and large radius, for a short time is completely equivalent to a small part of a "comoving" inertial reference frame (endowed with the same velocity). For all practical purposes the segment \(AB\) will belong to that inertial reference frame. But the velocities of light in the two directions \(AB\) and \(BA\) have to satisfy \((R)\). It follows that also the one way velocity of light relative to the comoving inertial frame cannot be \(c\) and must instead satisfy \((R)\).

The ET (of which the IT are a particular case) predict that the inverse one way velocity of light relative to the comoving system \(S\) is a linear function of \(\cos \theta\), where \(\theta\) is the angle between the light propagation direction and the absolute velocity \(\vec{v}\) of \(S\). Eq. \((10.5)\) applied to the cases \(\theta = 0\) and \(\theta = \pi\) easily gives a result compatible with \((R)\) only if \(e_1 = 0\). We thus see that our fundamental result \((R)\) is consistent with the physics of the inertial systems only if absolute simultaneity is adopted.

10. The rotating platform: \(e_1 = 0\)\(^{1710}\) parole

Next we review earlier results showing that the comparison between the relativistic description of rotating platforms and the physics of inertial reference systems points out to another fundamental difficulty. Furthermore we show that the difficulty can be overcome only by substituting the Lorentz transformations between inertial systems with the “inertial” transformations based on \(e_1 = 0\) \(^{[10-1]}\).

It is well known that no perfectly inertial frame exists in practice because of Earth rotation, of orbital motion around the Sun, of Galactic rotation. All knowledge about inertial systems has therefore been obtained in frames having small but non zero acceleration \(a\). For this reason the mathematical limit \(a \to 0\) taken in the theoretical schemes should be smooth and no discontinuities should arise between systems with small acceleration and inertial systems, because anyway our experiments are done in a reality in which \(a \neq 0\). This requirement will be shown not to be satisfied by the existing relativistic theory. We will show that a discontinuity arises between zero and very small acceleration. In cases of this type it would be wise to discard the zero acceleration limit and concentrate in the study of the predictions made by different theories at very small acceleration,
a region where all our experiments have been carried out, but of course just the opposite has been done under the influence of the great successes of the TSR.

Consider an inertial reference system $S_0$ and assume it is isotropic so that the one-way velocity of light relative to $S_0$ has the usual value $c$ in all directions. In relativity the latter assumption is true in all inertial frames, while in other theories only one frame satisfying it exists.

In a laboratory there is a circular platform (radius $r$ and center constantly at rest in $S_0$) which rotates uniformly around its axis with angular velocity $\omega$ and peripheral velocity $v = \omega r$. On its rim there is only a single clock $C_\Sigma$ (marking the time $t$). We assume it to be set as follows: When a clock of the laboratory momentarily very near $C_\Sigma$ shows time $t_0 = 0$ then also $C_\Sigma$ is set at time $t = 0$. When the platform is not rotating, $C_\Sigma$ constantly shows the same time as the nearby laboratory clocks. When it rotates, however, motion modifies the pace of $C_\Sigma$ and the relationship between the times $t$ and $t_0$ is taken to have the general form

$$t_0 = t F(v, ...)$$  \hspace{1cm} (10.1)

where $F$ is a function of velocity $v$ and eventually acceleration $a = v^2/r$ and higher derivatives of position (not shown). Eq. (10.1) is a consequence of the isotropy of $S_0$. Its validity can be shown in three elementary steps:

1. In the inertial system $S_0$ all directions are physically equivalent. If a clock is moving on a straight line $\ell$ with a certain speed $v$ relative to $S_0$, the change in the rate of advancement of its hands cannot depend on the orientation of $\ell$.

2. Similar is the case of the clock $C_\Sigma$ at rest on the rim of a uniformly rotating platform, with centre at rest in $S_0$. If $S_0$ is isotropical the rate of advancement of the $C_\Sigma$ hands cannot depend on the orientation of the clock instantaneous velocity vector, but only on speed $v$ and eventually acceleration.

3. This conclusion, clearly correct by symmetry reasons, was confirmed experimentally by the 1977 CERN measurements of the anomalous magnetic moment of the muon $[10-2]$. The decay of muons was followed very closely in different parts of the storage ring and the results showed a decay rate constant in the different points of the ring.

Thus we have every reason to believe (10.1) to be correct. We are of course far from ignorant about the function $F$. There are strong experimental indications
that the dependence on $a$ is totally absent and that $F(\nu, \ldots) = 1/R$, with $R$
given by (6.3). This is however irrelevant for our present needs as the results
obtained below hold for all possible factors $F$.

On the rim of the platform besides clock $C$$_\Sigma$ there is a light source
$\Sigma$, placed in a fixed position very near $C$$_\Sigma$. Two light flashes leave $\Sigma$ at the
time $t_1$ of $C$$_\Sigma$ and are forced to move on a circumference, by “sliding” on the internal
surface of a cylindrical mirror placed at rest on the platform, all around it and
very near its border. Mirror apart, the light flashes propagate in the vacuum. The
mirror behaves like a source ("virtual") and a source motion never changes the
velocity of the emitted light signals. Therefore the motion of the mirror cannot
modify the velocity of light. Thus, relative to the laboratory, the light flashes
propagate with the usual velocity $c$.

As seen already in the previous chapter the description of light propagation
given by the laboratory observers is the following: two light flashes leave $\Sigma$ at
time $t_{01}$. The first one propagates on a circumference, in the sense discordant
from the platform rotation, and comes back to $\Sigma$ at time $t_{02}$ after a full circle
around the platform. The second flash propagates on the same circumference, in
the sense concordant with the platform rotation, and comes back to $\Sigma$ at time $t_{03}$
after a full circle around the platform. These laboratory times, all relative to
events taking place in a fixed point of the platform very near $C$$_\Sigma$, are related to
the corresponding platform times via (10.1):

$$t_{0i} = t_i F(\nu, \ldots) \quad (i = 1, 2, 3).$$

The circumference length is assumed to be $L_0$ and $L$, measured in the
laboratory $S_0$ and on the platform, respectively. If $\tilde{c}(0)$ and $\tilde{c}(\pi)$ are the light
velocities, relative to the disk, for the flash propagating in the direction of the
disk rotation and in the opposite direction, respectively, one can show with a few
elementary steps using the very definition of velocity and (10.2):

$$\begin{align*}
\frac{1}{\tilde{c}(\pi)} &= \frac{L}{L_0} = \frac{L_0}{FL} - \frac{L_0}{L_0} = \frac{L_0}{FL} \frac{1}{c + \nu} \\
\frac{1}{\tilde{c}(\pi)} &= \frac{L_0}{L_0} = \frac{L_0}{FL} - \frac{L_0}{L_0} = \frac{L_0}{FL} \frac{1}{c - \nu}
\end{align*}$$

where (9.2) and (9.4) were used. From (10.3) it follows, with $\beta = \nu / c$: 
\[
\frac{\tilde{c}(\pi)}{\tilde{c}(0)} = \frac{1 + \beta}{1 - \beta}
\]  

(10.4)

Notice that the functions \( F, L, L_0 \) have disappeared in the ratio (10.4).

Next comes an important remark. Clearly, Eq. (10.4) gives us not only the ratio of the two global light velocities for full trips around the platform, but the ratio of the instantaneous velocities as well. In fact the isotropy of the inertial system \( S_0 \) ensures, by symmetry, that the instantaneous velocities of light are the same in all points of the rim of the rotating circular disk whose center is at rest in \( S_0 \). There is no reason why the light instantaneous velocities relative to the disk in the different points of the rim should not be equal to one another [10-3]. With reference to Fig. 11 we can therefore write the equations

\[
\tilde{c}_{\phi_1}(0) = \tilde{c}_{\phi_2}(0) ; \quad \tilde{c}_{\phi_1}(\pi) = \tilde{c}_{\phi_2}(\pi)
\]

where \( \phi_1 \) and \( \phi_2 \) are arbitrary values of the angle \( \phi \).

Therefore the light instantaneous velocities relative to the disk will also coincide with the average velocities \( \tilde{c}(0) \) and \( \tilde{c}(\pi) \), and Eq. (10.4) will apply also to the ratio of the instantaneous velocities [thus we do not need a different symbol for the instantaneous velocities].

![Figure 11](image.png)

The result (10.4) holds with the same numerical value for platforms having different radius, but the same peripheral velocity \( v \). Let a set of circular
platforms be given with centres at rest in $S_0$. Let their radii be $r_1$, $r_2$, ... $r_i$, ..., with $r_1 < r_2 < ... < r_i < ...$, and suppose they are made to spin with angular velocities $\omega_1$, $\omega_2$, ... $\omega_i$, ... such that $v$ is constant and $\omega_1 r_1 = \omega_2 r_2 = ... = \omega_i r_i = ... = v$. Obviously, then, (10.4) applies to all such platforms with the same $\beta$ ($\beta = v / c$). The centripetal accelerations decrease regularly with increasing $r_i$. Therefore, a small part $AB$ of the rim of a platform, having peripheral velocity $v$ and large radius, for a short time is completely equivalent to a small part of a "comoving" inertial reference frame (endowed with the same velocity). For all practical purposes the segment $AB$ will belong to that inertial reference frame. But the velocities of light in the two directions $AB$ and $BA$ have to satisfy (10.4). It follows that the one way velocity of light relative to the comoving inertial frame cannot be $c$ and must instead satisfy

$$\frac{c_1(\pi)}{c_1(0)} = \frac{1 + \beta}{1 - \beta}$$

(10.5)

As we saw in chapter 7, the equivalent transformations (of which the inertial transformations are a particular case) predict the inverse one way velocity of light relative to the comoving system $S$:

$$\frac{1}{c_1(\theta)} = \frac{1}{c} + \left[ \frac{\beta}{c} + e_1 R \right] \cos \theta$$

(10.6)

where $\theta$ is the angle between the light propagation direction and the absolute velocity $\vec{v}$ of $S$. Eq. (10.6) applied to the cases $\theta = 0$ and $\theta = \pi$ easily gives

$$\frac{c_1(\pi)}{c_1(0)} = \frac{1 + \beta + c e_1 R}{1 - \beta - c e_1 R}$$

(10.7)

Clearly, Eq. (10.7) is compatible with (10.4) only if $e_1 = 0$. We can see that also our result (10.4) is consistent with the physics of the inertial systems only if absolute simultaneity is adopted. For a better understanding of the reasons why the TSR does not work consider again the
The ratio \( \rho = \tilde{c}(\pi) / \tilde{c}(0) \) plotted as a function of acceleration \( a \) for rotating platforms of constant peripheral velocity and decreasing radius (increasing acceleration). The prediction of the TSR is \( \rho = 1 \) for \( a = 0 \) (black dot on the \( \rho \) axis) and is not continuous with the \( \rho \) value of the rotating platforms.

\[
\rho \equiv \frac{\tilde{c}(\pi)}{\tilde{c}(0)} \tag{10.8}
\]

which, owing to (10.5), is larger than unity. Therefore the light velocities parallel and antiparallel to the disk peripheral velocity are different. For the TSR this conclusion is unacceptable, because a set of platforms, all endowed with the same peripheral velocity locally approximates an inertial system better and better with increasing radius. The logical situation is shown in Fig. 12.

Thus the TSR predicts for \( \rho \) a discontinuity at zero acceleration. While all the experiments are performed in the real physical world [where of course \( a \neq 0 \), \( \rho = (1 + \beta) / (1 - \beta) \)], the theory has gone out of the world \( (a = 0, \quad \rho = 1) \)!

Notice that the velocity of light given by Eq. (10.6) with \( e_1 = 0 \) is required for all inertial systems but one, the isotropical system \( S_0 \). In fact, for every small region \( AB \) of every such system it is possible to imagine a large rotating platform with center at rest in \( S_0 \) and rim locally comoving with \( AB \) and the
result (10.6) can be applied. Therefore the velocity of light depends on direction in all inertial systems with the exception of the privileged one, $S_0$.

We conclude by stressing that absolute simultaneity is a logical necessity not only in relativistic physics, but also in the broader domain of the “general transformations” given by Eq. (7.1). In fact it has been proved [10-3] that $e_1 = 0$ is a characteristic property of all theories treating inertial systems in a way continuous with the accelerated systems. In view of our results absolute simultaneity seems to be the only serious way of dealing with space and time physics. Seen in this way, absolute simultaneity looks very much like a fundamental property of nature.

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Chapter 11

Linear accelerations: \( e_1 = 0 \)

The hypothetical indifference of the physical reality with respect to clock synchronization exists only insofar as one neglects accelerations. An accelerating body can be considered at rest in different inertial systems during infinitesimally small time intervals, and it is therefore impossible to adopt in those systems a procedure, such as Einstein’s, requiring a finite time to be implemented. Nevertheless physical events take place and a form of objective synchronization must somehow be fixed by nature itself. Essentially, this is what we saw with rotating disks. We will now consider the linear accelerations to discover that also in this case the so called absolute synchronization, corresponding to \( e_1 = 0 \), establishes itself without human intervention.

The spaceships \( A \) and \( B \) are initially at rest in the (privileged) inertial system \( S_0 \). Their clocks are synchronous with those of \( S_0 \). At time \( t_0 = 0 \) they start accelerating, and they do so in the same identical way, having the same velocity \( v(t_0) \) at every time \( t_0 \) of \( S_0 \), until, at a time \( t_0 \) of \( S_0 \), they reach a given velocity \( v = v(t_0) \); for all \( t_0 \geq t_0 \) the spaceships remain at rest in an inertial system \( S \), which they concretely constitute.

The transformation relating \( S_0 \) and \( S \) is necessarily the inertial one, if no final clock re-synchronization is applied correcting what nature generated during the acceleration. Proof: Since \( A \) and \( B \) accelerate in the same way, their clocks accumulate the same delay with respect to those at rest in \( S_0 \). Therefore two events, in different points, simultaneous in \( S_0 \) will be such also in \( S \). Clearly we have a case of absolute simultaneity and the condition \( e_1 = 0 \) must hold.

Our spaceships \( A \) and \( B \), initially at rest in \( S_0 \), have clocks aboard \( (C_A \) and \( C_B \), respectively) synchronized with the clocks of \( S_0 \). Before departure \( (t_0 = 0) \) \( C_A \) and \( C_B \) are used to measure the velocity of a pulse of light propagating from \( A \) to \( B \). Of course the result is bound to be the usual value of the TSR, \( c \), because before departure the inertial system of \( A \) and \( B \).
is $S_0$ and in $S_0$, clocks (including $C_A$ and $C_B$) have been synchronized with the Einstein method.

The same experiment is repeated when the spaceships are at rest in $S$. Now, if the invariance of the velocity of light were a law of nature, one should find the same in $S$ and in $S_0$, given that the retardation of $C_A$ and $C_B$ during the accelerated motion has been exactly the same. Instead, the velocity of light in $S$ from $A$ to $B$, turns out to be different from $c$. Therefore everything happens as if we measured the velocity of light with two clocks, then set backwards their hands by the same amount, then measured again the velocity of light and found a different result. It is a surprise!

This result can only be understood in terms of inequivalence of the inertial systems, in the sense that the velocity of light relative to an inertial system depends on the absolute motion of the latter.

### 11. Linear accelerations: $e_1 = 0$

The hypothetical indifference of the physical reality with respect to clock synchronization exists only insofar as one neglects accelerations. In fact, when a body is accelerating, one can consider it at rest in different inertial systems during infinitesimally small time intervals, and it is therefore impossible to adopt in those systems a procedure, such as Einstein’s, requiring a finite time to synchronize clocks placed in different points. Nevertheless physical events take place and synchronization must somehow be fixed by nature itself. Essentially, this is what we saw in the two previous sections with rotating disks. We will now see how this happens with linear accelerations and we will discover that also in this case the so called absolute synchronization arises spontaneously without human manipulations of clocks [11-1]. With our notation this corresponds to the choice $e_1 = 0$.

The argument to be discussed in this chapter concerns two spaceships $A$ and $B$ moving in space on the same straight line parallel to their (common) velocity. For us they will be structureless material points as we are not interested in the effects of the Lorentz contractions on the distances inside a spaceship. Interesting for our tasks are instead the consequences of motion on the clocks, on the relative distance and on the velocity of $A$ and $B$. The argument could easily be generalized to more complex structures, networks of
material points organized in 2 or 3 dimensions, but we will limit the discussion to the most elementary case.

Two identical spaceships $A$ and $B$ are initially at rest on the $x_0$ axis of the (privileged) inertial system $S_0$ at a distance $d_0$ from one another. Their clocks are synchronous with those of $S_0$. At time $t_0 = 0$ they start accelerating in the $+x_0$ direction, and they do so in the same identical way, in such a way as to have the same velocity $v(t_0)$ at every time $t_0$ of $S_0$, until, at a time $t_0 = \tilde{t}_0$ of $S_0$, they reach a given velocity $v = v(\tilde{t}_0)$ parallel to $+x_0$; for all $t_0 \geq \tilde{t}_0$ the spaceships remain at rest in a new inertial system $S$, which they concretely constitute. In $S_0$ the positions of $A$ and $B$ at any time $t_0 \geq \tilde{t}_0$ are:

\[
x_{0A}(t_0) = x_{0A}^0 + \int_0^{t_0} dt_0' v(t_0') + (t_0 - \tilde{t}_0)v
\]

\[
x_{0B}(t_0) = x_{0B}^0 + \int_0^{t_0} dt_0' v(t_0') + (t_0 - \tilde{t}_0)v
\]

so that

\[
x_{0B}(t_0) - x_{0A}(t_0) = x_{0B}(0) - x_{0A}(0) = d_0
\]

Eq. (11.2) implies that the motion of $A$ and $B$ does not modify the distance $d_0$ between the spaceships as seen from $S_0$. The same distance seen from $S$ (call it $d$) instead increases during acceleration, as the unit-rod measuring it undergoes a progressive contraction. One has:

\[
d = \frac{d_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

In fact the observer in $S_0$ will check: (i) That the distance $d_0$ between $A$ and $B$ remains the same after the acceleration, as shown by (11.2); (ii) That the unit rod in $S$ is Lorentz contracted if compared with a similar rod at rest in $S_0$; (iii) That the observer in $S$ using his shortened rod to measure the $A$-$B$ distance finds a value larger by a factor $1/\sqrt{1 - \frac{v^2}{c^2}}$ than found before departure when the spaceships were at rest in $S_0$. This measurement is an
objective procedure and its result (= number of times the rod of $S$ fits into the $A-B$ distance) cannot depend on the subjective point of view. Therefore the observer in $S$ finds indeed what the observer in $S_0$ sees him finding, namely a distance between $A$ and $B$ larger by a factor $1 / \sqrt{1-\frac{v^2}{c^2}}$, as given by (11.3).

It will next be shown that the transformation relating $S_0$ and $S$ is necessarily the inertial one, if no final clock re-synchronization is applied correcting what nature itself generated during the acceleration of the two spaceships. Since $A$ and $B$ accelerate exactly in the same way, their clocks will accumulate exactly the same delay with respect to those at rest in $S_0$. Motion is the same for $A$ and $B$; all effects of motion will necessarily coincide, in particular time delay. Therefore two events simultaneous in $S_0$ will be such also in $S$, even if they take place in different points of space. Clearly we have a case of absolute simultaneity and the condition $e_1 = 0$ must hold in (7.2), reducing these transformations to their inertial form (7.5).

In order to make the point as clear as possible we check now that the velocity in $S$ of a light pulse traveling from $A$ to $B$ when the two spaceships are at rest in $S$ (while, of course, they move with velocity $v$ with respect to $S_0$) is consistent with the inertial velocity of light formula (7.6). Let a light signal leave $B$ at time $t_A$ and reach $B$ at time $t_B$, both times being measured in $S_0$. Its velocity $\tilde{c}$ in $S$ is by definition

$$\tilde{c} = \frac{d}{t_B - t_A}$$

where $d$ is given by (11.3). Now define:

$\tau$: delay generated by velocity up to time $\tilde{t}_0$ at which acceleration stops;

$t_{0A}$: time of emission of light signal from $A$ as seen in $S_0$ ($t_{0A} > \tilde{t}_0$);

$t_{0B}$: time of arrival of light signal at $B$ as seen in $S_0$ ($t_{0B} > t_{0A}$).

Since for all $t_0 \geq \tilde{t}_0$ time dilation in $S$ is due to the constant velocity $v$, one has:
\[
\begin{align*}
\tau_A &= t - t_0 + (t_0 - t) \sqrt{1 - \frac{v^2}{c^2}} \\
\tau_B &= t - t_0 + (t_0 - t) \sqrt{1 - \frac{v^2}{c^2}}
\end{align*}
\]  
(11.5)

By subtracting the first equation from the second, one gets

\[\tau_B - \tau_A = (t_0 - t) \sqrt{1 - \frac{v^2}{c^2}}\]  
(11.6)

Eq. (11.6) is the clock retardation formula for the travel time of the light pulse.

The point \(x_{0B}\) of \(S_0\) where the light is absorbed by \(B\) must satisfy:

\[
\begin{align*}
\tilde{x}_{0B} &= x_{0A} + c(t_0 - t_0A) \\
\tilde{x}_{0B} &= x_{0B} + v(t_0 - t_0A)
\end{align*}
\]  
(11.7)

if \(x_{0A}\) and \(x_{0B}\) are the positions of the spaceships \(A\) and \(B\) respectively at time \(t_0A\) of emission of the light signal. In fact, while the light signal goes from \(x_{0A}\) to \(x_{0B}\) with velocity \(c\), spaceship \(B\) moves from \(x_{0B}\) to \(\tilde{x}_{0B}\) with velocity \(v\). From (11.7) it follows:

\[x_{0B} - x_{0A} = c(1 - \frac{v}{c})(t_0 - t_0A)\]  
(11.8)

Remembering that (11.2) holds for all times \(t_0\) and using (11.3) and (11.6), Eq. (11.8) gives

\[
\frac{t_B - t_A}{d} = \frac{1 + \frac{v}{c}}{c}\]  
(11.9)

which compared with (11.4) gives finally:

\[
\tilde{c} = \frac{c}{1 + \frac{v}{c}}\]  
(11.10)
Therefore the velocity of light in $S$ satisfies (7.6) with $\theta = 0$. This is what one expects from the inertial transformation since the straight line connecting the spaceships $A$ and $B$ has been assumed parallel to their velocity.

These considerations show with clarity the probable point of arrival of this research, a new theory in which the slowing down of clocks is no longer relative, but only dependent on velocity with respect to the privileged frame. The existence of a vacuum endowed with concrete physical properties becomes acceptable. Naturally, one needs also to verify the new theory experimentally, but in a certain sense this has already been done, at least in the case of the Sagnac effect [11-2].

Not only the absolute simultaneity is concretely realised in the moving frame of the two spaceships, but one can find other convincing arguments showing that it gives the most natural description of the physical reality. We will suppose that our spaceships have passengers $P_A$ and $P_B$, who are homozygous twins. Of course in principle nothing can stop them from re-synchronizing their clocks once they have finished accelerating and the two spaceships are at rest in $S$. If they do so, however, they find in general to have different biological ages at the same (re-synchronized) $S$ time, even if they started the space trip at exactly the same $S_0$ time and with the same velocity, as stipulated above. Everything is regular, instead, if they do not operate any asymmetrical modification of the time shown by their clocks.
Figure 13. Two identical spaceships \( A \) and \( B \) are initially at rest on the \( x_0 \) axis of the inertial system \( S_0 \). After having accelerated in exactly the same way \( A \) and \( B \) are at rest in a different inertial system \( S \) which they concretely constitute.

In fact we already concluded that clocks in \( A \) and \( B \) are retarded in the same way, and that the transformations \( S - S_0 \) must be the inertial ones. Also the ageing of the twins must have been the same, since at every time before, during and after the acceleration they were in identical physical conditions. Therefore the twins have the same age when the times shown by their clocks are the same if they have been synchronized in \( S_0 \) before departure and never modified after. Naturally \( P_A \) and \( P_B \) can inform one another of their biological ages (e.g., via telefax) by exchanging pictures in which the times they were taken is marked: the twin receiving a picture can check in his archives that at the time shown on his brother’s picture he had exactly the same look, and therefore the same age.

Remember that our spaceships \( A \) and \( B \), initially at rest on the \( x_0 \) axis of the isotropic inertial system \( S_0 \) at a distance \( d_0 \) from one another, were supposed to have clocks aboard synchronized with the clocks of \( S_0 \). Let \( C_A \)
and \( C_B \) be two clocks aboard \( A \) and \( B \), respectively. Suppose that before the
time of departure \((t_0 = 0)\) \( C_A \) and \( C_B \) are used to measure the velocity of a
pulse of light propagating from \( C_A \) to \( C_B \). Well, of course the result is bound
to be the usual value of the TSR, \( c \), because before departure the inertial
system of \( A \) and \( B \) coincides with \( S_0 \) and in \( S_0 \), presumed to be an
isotropic system, clocks (including \( C_A \) and \( C_B \)) have been synchronized with
the Einstein method.

The same experiment is repeated after the acceleration has ended and
the spaceships are at rest in the different inertial system \( S \). Now, if the
invariance of the velocity of light were a law of nature one should find the
same result in \( S \) and in \( S_0 \), given that the retardation of \( C_A \) and \( C_B \) during the
accelerated motion is exactly the same. Instead, as we saw, the velocity of light
in \( S \) from \( A \) to \( B \), turns out to be given by (11.10). Notice that the equal
retardation of \( C_A \) and \( C_B \) is expressed by the equality of the proper times of
\( C_A \) and \( C_B \) and is therefore an objective property on which all observers
should agree. Therefore everything seems to go as if we measured the velocity
of light with two clocks, then set backwards their hands by the same amount,
then measured again the velocity of light and found a different result. It is a
surprise!

This result can only be understood in terms of inequivalence of the
inertial systems, in the sense that the second postulate of the theory of
relativity does not hold in nature and the velocity of light relative to an inertial
system depends on the absolute motion of the latter. This seems to imply the
existence of an objectively privileged system.

Naturally \( P_A \) and \( P_B \) can use a different synchronization of clocks, if
they wish, e.g. Einstein's synchronization leading to the validity of the \( S - S_0 \)
Lorentz transformations. To do so they must send a light signal, e.g. from \( A \) to
\( B \), and they must reset at least one clock. We can even suppose that every twin
has two clocks and keeps the first one set on absolute time, while regulating
the second to show the Einstein-Lorentz time. More exactly we assume that:
$P_A$ has a first clock $C_A$ marking the natural time $t_A$

$P_A$ has a second clock $\bar{C}_A$ marking the E-L time $\hat{t}_A$

$P_B$ has a first clock $C_B$ marking the natural time $t_B$

$P_B$ has a second clock $\bar{C}_B$ marking the E-L time $\hat{t}_B$

After a certain initial time interval during which $t_A = \hat{t}_A = t_B = \hat{t}_B$ only $\bar{C}_B$ is resynchronized in the following way. At a certain given time a light signal is sent from $A$ to $B$. The convention that the one-way velocity of light in $S$ is $c$ forces the observer in $B$ to rotate the hands of his clock $\bar{C}_B$ in such a way that the time necessary for the signal to cover the distance $d$ from $A$ to $B$ be measured to be $d/c$.

Clearly, after re-synchronizing, at a given time $t_0$ of $S_0$ one will have $t_B = t_A$, but $\hat{t}_B \neq \hat{t}_A$. The simultaneity of $\bar{C}_A$ and $\bar{C}_B$ is now different from that of $T_A$ and $T_B$! If $P_A$ and $P_B$ exchange pictures of themselves in which also the times marked by the clocks $C$ and $\bar{C}$ are shown, they discover having had the same age at the same time $t$, but different ages at the same time $\hat{t}$. This provides a strong argument in favor of the inertial transformations, because not all natural “clocks” can be synchronized: irreversible processes exist such as the ageing of $P_A$ and $P_B$! Nature itself favors the inertial transformations from $S_0$ to $S$ for describing the time of inertial systems concretely produced.


Chapter 12

Overcoming the block universe:

In a previous chapter we said already that Karl Popper, in his autobiography, was critical of relativistic determinism. He recalled a discussion he had with Einstein in Princeton (1950): "I tried to persuade him to give up his determinism, which amounted to the view that the world was a four-dimensional Parmenidean block universe in which change was a human illusion, or very nearly so. (He agreed that this had been his view, and while discussing it I called him 'Parmenides')."

It is interesting to know that Einstein really believed in the existence of the “block universe” of relativity. In early 1955 Michele Besso, his friend of a lifetime, died. Einstein wrote a letter of condolences to the sons and the sister of Besso, in which the description of the block universe was offered as a consolation. His words were: “Again he went before me by little in leaving this strange world. But this is not meaningful. For us, militant physicists, this separation between past, present and future has only the value of an illusion, however tenacious it may be.” Let us try to understand the point. Einstein here says that it is wrong to be sorry at a person’s death, just as it would be wrong to feel bad because we extend physica[lly in space for 2 meters or so. Our life-time is nothing but our life-(fourth dimension of space). Our body extends not only in the traditional three dimensions: from feet to head; from belly to back; from right hand side to left hand side; but also in the fourth dimension of space, from birth to death. Therefore Michele is not dead in the human sense of the term. He occupies his personal volume in the four dimensions as he “always” did and as he “forever” will. He is standing out there and – who knows? maybe is smiling of our sorrow for his “death.” Change the words, adopt another style, but this is the way Albert Einstein thought of “death,” a few months before his physical limit in the fourth dimension would give a proof of its concreteness.
The theory of relativity leads to unpleasant consequences if used to understand how objective reality should be described. To begin with, what we see cannot be considered real in the present, because by looking at distant objects we do not see them as they are now, but as they were when the light now entering our instruments left them. Also, it is not reasonable to attribute reality to the future, because good sense tells us that the future does not yet exist and that it is at least partly undetermined. For these reasons a reasonable definition of reality seems to be the following: all what exists now, here and elsewhere. A different choice would define real either things that do not exist anymore, or things that do not yet exist. The light from a galaxy can require hundreds of millions of years to reach our instruments, and in this long time the object that emitted it could have dissolved, have collided with another cosmic object, or have exploded (there are pictures of galaxies devastated by huge explosions). It will, anyway, have evolved, and could now differ significantly from what we observe. It seems safe to conclude that reality is not what we see.

Unfortunately, even if we start with a reasonable approach to reality, the relativistic theory forces us to accept a predetermined future. Let us adopt the relativistic description, with a Minkowski diagram having space in abscissa (only one dimension for graphical reasons!) and time in ordinates. At time \( t = 0 \) an observer \( U_0 \) located in the origin of an inertial system \( S \) must regard as being objectively real down to the smallest detail all events in space. In this diagram space is represented as \( x \) axis, whose equation is \( t = 0 \), and which therefore contains all events simultaneous with the instantaneous presence of \( U_0 \) in the origin at \( t = 0 \).

If we consider a different inertial reference frame \( S' \), its axes \( x' \) and \( t' \) are represented in the Minkowski diagram as straight lines in the plane \((c t, x)\) because of the linearity of the Lorentz transformations. The observer \( U'_0 \) at rest in \( S' \) has the right to attribute reality to all events happening at his present time \( t' = 0 \). The set of these events is of course different from the set of events constituting the reality of \( U_0 \). According to the relativity principle it does not make any sense to ask which one of the two observers \( U_0 \) and \( U'_0 \) is correct. Given the complete symmetry between inertial systems, they are both correct. So all the events on the \( x' \) axis, whose equation is \( t' = 0 \) and whose inclination depends on the velocity of \( S' \) relative to \( S \), can be considered just as real as the
events on the $x$-axis. The Lorentz transformation of time from $(ct, x)$ to $(ct', x')$ reads

$$ct' = \frac{ct - xv/c}{\sqrt{1 - v^2/c^2}}$$

(12.1)

where $v$ is the relative velocity of the $S$ and $S'$ systems. If we set $ct' = 0$ the equation becomes

$$ct = (v/c)x$$

(12.2)

a straight line with a slope $(v/c) < 1$. Therefore, the reality line of the observer $U'_0$ has an inclination in time with respect to that of $U_0$ and also passes through the origin of the Minkowski diagram (see Fig. 14). Clearly $U'_0$ will attribute reality to events in $U_0$'s future, which are therefore not part of $U_0$'s present reality. In the previous example, however, these future events are elsewhere and do not belong to the personal future of $U_0$, who is assumed at rest in the origin $x = 0$.

The meaning of the previous argument can easily be understood by assuming that in $S$ there are several observers $U_1, U_2, ... U_n$ placed in different points $x_1, x_2, ... x_n$ of the $x$ axis, all provided with clocks synchronized with the Einstein procedure. These observers are all equivalent in their description of reality, since time $t = 0$ is the same for all of them, and reality consists of the events placed on the $x$ axis; naturally they are also equivalent to the observer $U_0$ in $x = 0$ considered before. It is now clear that the reality line of $U'_0$ passes through the personal future of some of the observers at rest in $S$ [those placed in points having positive (negative) $x$ if $S'$ moves with velocity in direction $+x(-x)$]. All this attributes reality to the future of some individual observer of $S$, e.g., to the future of $U_1$.

I can think to be $U_0$ and to receive in this moment from the observer $U'_0$ (who is passing near me at high velocity) a message containing the information that the reality (for me, future) of my brother $U_1$ is “already” perfectly determined. It is however difficult for me to believe that this message tells the truth, as I know that the future is all to be built and is only very partially predetermined in the present reality. My brother $U_1$ is a free man who has the
faculty of choice and is certainly capable to fix as he wishes at least some features of his future.

Figure 13. In a Minkowski diagram the reality line of a moving observer is the $x'$ axis which includes events such as $E_1, E_2, ...$ belonging to the future of the observers $U_1, U_2, ...$ placed on the $x$ axis of the "stationary" observer $U_0$.

Reality has so far been attributed to a single instant of the future reality only, but the argument can easily be generalized. Indeed infinitely many reality lines pass through every point of the diagram $(ct, x)$, each such line representing the (relativistic) reality of some legitimate inertial observer. The only restriction is the inclination of these reality lines in a diagram $(ct, x)$: it can never exceed $\pi / 4$, since all velocities are subluminal. Nevertheless past, present, and future are completely real, that is pre-established in the minutest detail. Passing from two to four dimensions according to the TSR we can conclude that the whole space-time $(ct, x, y, z)$ is real, despite the different perception humans have of it. In other words my future is real, i.e. fixed in the tiniest details, despite its looking to me as largely undetermined, unshaped, presently unreal.

It is interesting to stress that Einstein really believed in the existence of the block universe of relativity. In early 1955 Michele Besso, his friend of a
lifetime, died. Einstein wrote a letter of condolences to the sons and the sister of Besso, in which the description of the block universe was offered as a consolation. His words were: “Again he went before me by little in leaving this strange world. But this is not meaningful. For us, militant physicists, this separation between past, present and future has only the value of an illusion, however tenacious it may be.” [12-1]

Karl Popper, in his autobiography, was critical of the relativistic determinism. He recalled a discussion he had with Einstein in Princeton (1950): "I tried to persuade him to give up his determinism, which amounted to the view that the world was a four-dimensional Parmenidean block universe in which change was a human illusion, or very nearly so. (He agreed that this had been his view, and while discussing it I called him 'Parmenides')." [12-2]

Popper's identification is justified, since for both Einstein and Parmenides the subjective impression of evolution is pure appearance. Popper found this description of reality unacceptable, and it is difficult to disagree with him. We will soon see what must be dropped to get rid of the hyperdeterministic universe: the strong formulation of the relativity principle with the consequential conventionality of simultaneity.

Thus relativity leads to a very strange conception of the universe, in which a single reality fills uniformly past, present, and future: at this present time other observers no less legitimate than I consider my personal future as given in all detail. According to them there is not the slightest freedom which I can use in order to influence the course of events. The impression I have of a reality evolving sometimes in a casual (non deterministic) way, would therefore be entirely subjective, a limitation (due to my poor means of observation) to a fixed time section of the complete fourdimensional reality. Relativity therefore leads one to accept a hyperdeterministic universe in which the whole future is completely pre-established in the minutest details and in which all sensations of individual freedom (even those limited to very simple events, like the choice between holding and dropping a stone) are pure illusions.

The previous argument is founded on the idea that every observer is right in considering real all what exists around him and elsewhere at his present time. There is of course another possibility compatible with relativism, based on the idea that all observers are wrong and that no reality exists outside the thinking subjects. In such a case the plane \((c t, x)\) would become a "tabula rasa", completely empty, and the corresponding philosophy would be that of the purest idealism. Such a 'solution' is obviously even less interesting than the last one. It does not seem possible to escape from this vicious circle
(hyperdeterminism, idealism) without abandoning that formulation of the relativity principle which leads necessarily to the Lorentz transformations.

On the other hand one can stress that the predetermination of the future reality depends only on the relativistic definition of simultaneity, that is on a conventional aspect of the theory. A suitable reformulation should allow a more reasonable description of the future without changing the excellent agreement of the theory with the empirical evidence. In fact, the most general inertial transformation for time from \( S \) to \( S' \), the two systems moving with respect to the privileged system with velocities \( v \) and \( v' \), respectively, is

\[
c t' = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{v'^2}{c^2}}} c t
\]

(12.3)

with the \( x' \) axis (reality line) of \( S' \). Therefore, all the observers agree about the identification of a single reality line of the universe. That is, they agree about the present, or, which is the same, about which events elsewhere are simultaneous with the here-now. Obviously, the inertial transformations allow a much more reasonable description of the future, which does not appear anymore to be predetermined, as it happen) is thus evident also from the elimination of the block universe paradox.

On the other hand one can stress that the predetermination of the future reality depends only on the relativistic definition of simultaneity, that is on a conventional aspect of the theory. A suitable reformulation should allow a more reasonable description of the future with no change of the excellent agreement of the theory with the empirical evidence. The most general “equivalent” transformation of time from \( S_0 \) to \( S \) is given by (7.2):

\[
t = Rt_0 + e_1(x_0 - vt_0)
\]

(12.4)

Obviously, \( c t = 0 \) implies

\[
t_0 = \frac{-e_1}{R - e_1v} x_0
\]

(12.5)

At time \( t = 0 \) an observer \( U_0 \) located in the origin of \( S_0 \) regards as objectively real down to the smallest detail all events in space. In this diagram space is the \( x \) axis (equation \( t = 0 \)), and contains all events simultaneous with the presence
of $U_0$ in the origin at $t = 0$. The reality line of the observer $U'_0$ has an inclination in time with respect to that of $U_0$ and also passes through the origin of a Minkowski-type diagram. Clearly, the reality in the future argument of the TSR can be repeated here without difficulty, and the result is again the appearance of the block universe. As we saw there is an exception with the case $e_1 = 0$ which gives the inertial transformations, for which all the observers agree about the identification of a single reality line of the universe. That is, they agree about the present, or, which is the same, about which events elsewhere are simultaneous with the here-now. Obviously, the inertial transformations allow a much more reasonable description of the future, which does not appear anymore to be predetermined, as it happened with the Lorentz transformations. The superiority of the inertial transformations based on absolute simultaneity is thus evident also in the elimination of the block universe paradox.

[12-1] [EB, p. 312]
[12-2] [UQ, p. 129]
Chapter 13

The aberration of starlight: $e_1 = 0$

The phenomenon of aberration of the starlight, discovered by Bradley in 1725, is very important in relativistic physics, so much that Einstein discussed it in his first article on the TSR. Aberration is relevant also to the theory presented in this book. Given that the TSR is clearly in trouble in trying to explain aberration, we will check that the alternative theory of the inertial transformations provides a completely satisfactory interpretation of the phenomenon.

From the angular deviation of the light of a star, observed during a year, it is possible to deduce the velocity of light. But the light of the star follows a one way path towards the Earth, a fact which could lead people to believe that aberration allows one to measure the one way velocity of light. Actually it is not so, as all the equivalent transformations predict exactly the same aberration angle, even though the one way velocity is different for different equivalent transformations. The measured velocity of light is $c_2$.

No star is really where we see it, because the direction from which the light seems to be coming is bent by the motion of our planet in a way similar to the bending of rain for the driver of a car. Maybe the rain falls vertically, but the driver will see it coming at an angle from the forward direction: It is known that the Earth has two important motions: (1) At the velocity 30 km/sec there is the circumsolar orbital motion; (2) At the velocity 300 km/sec the Earth, as part of the solar system and of the local arm of the galaxy, revolves around the center of the Milky Way. The second motion would seem to be more important, but it gives rise to unobservable effects because the tour around the galaxy center takes about two hundred thousand years to be accomplished. A time of some centuries in practice is so small that the piece of galactic orbit is at all effects rectilinear. This produces an aberration, but since we do not know the true position of any star, we are unable to detect it. We can only say where an object of the sky is seen, but this is not enough for a determination of aberration. Notice the difference between the two movements. In circumsolar orbital motion the Earth centripetal acceleration is very important. The same conclusion does not hold for the movement around the Milky Way center, which in practice is unobservable. When accelerations are in play the observability of aberration becomes concrete. The aberration of starlight due to orbital motion of the Earth is easy to observe, because the fixed stars change their positions in the sky in a systematic way with a period of one year.

In this chapter the difficulties of the standard relativistic theory in the explanation of aberration are reviewed. It so happens that these difficulties are not shared by the TIT which provides a natural description of the phenomenon. In short, the important role of acceleration seems once more to weaken the possibility that the standard relativistic theory describes correctly the real phenomenon.
The aberration of starlight: $e_1 = 0$

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Consider the propagation of a localized light pulse $P$ from the point of view of the privileged reference system $S_0$ of the equivalent theories, relative to which the velocity of light is the same in all directions. If $\theta_0$ is the inclination with respect to the $x_0$ axis of the trajectory of $P$ and $\theta$ is the inclination of the same trajectory as judged in $S$, one can prove [13-1] an aberration formula mathematically identical to the one of relativity, namely:

$$\tan \theta = \frac{c R \sin \theta_0}{c \cos \theta_0 - v}$$  \hspace{1cm} (13.1)

where $v$ is the velocity of $S$ measured in $S_0$ and $R = \sqrt{1 - v^2 / c^2}$ as in (6.3). The quantity $c$ in (13.1) is the one way speed of light relative to $S_0$, and, at the same time, the two way speed of light relative to all inertial reference systems.

Therefore, all the quantities entering in the right hand side of (13.1) are relative to the isotropic system $S_0$ for which all equivalent theories accept the same value of the velocity of light, and thus the same synchronization of clocks. Clearly, all the equivalent transformations (among which there are Lorentz’s) agree on the numerical values of $\theta_0$ and $v$. Therefore, thanks to (13.1), for any given reference system $S$ they predict the same value of the aberration angle $\theta$.

Although we are presently unable to identify the privileged inertial system $S_0$, the previous conclusion is obviously enough for saying that once given (13.1), we have obtained a complete explanation of aberration from the point of view of the equivalent transformations, based on the existence of a privileged system: if the absolute aberration angle of a star is the same for all $S$, also the relative
aberration angle observed between two moving systems $S$ and $S'$ has to be the same!

Figure 14. A localized light pulse $P$ propagates in the “stationary” inertial system $S_0$, relative to which the velocity of light is the same in all directions. One seeks the description of the motion of $P$ relative to the “moving” inertial system $S$.

Aberration explained in terms of absolute motion, as presented above, provides the resolution of a longstanding problem of the relativistic approach. Einstein deduced the aberration formula (13.1) from the idea that $v$ is the “velocity of the observer relatively to an infinitely distant source of light.” [13.2] This idea was repeated by many authors, clearly because the use of the relative velocity is the most natural thing to do in a theory, such as the TSR, based on relativism. For example Møller stated that the phenomenon called aberration consists of the fact that “the direction of a light ray depends essentially on the velocity of the light source relative to the observer.” [13.3]

If, however, we imagine the stars as molecules of a gas in random motion, we have to admit that the velocity relative to the Earth varies from star to star. This conclusion contradicts the fact that the observed angle of aberration is the same for all stars. In 1950 Ives stressed that “The idea sometimes met with that aberration ... may be described in terms of the relative motions of the bodies concerned, is immediately refuted by the existence of spectroscopic binaries with
velocities comparable with that of the earth in its orbit. These exhibit aberrations no different from other stars.” [13,4]

The components of such binary systems at some times can have velocities relative to the Earth very different from one another; nevertheless it is well known that these components exhibit always the same aberration angle, by the way not different from that of single stars. The argument was developed by Eisner in 1967: A distant observer \( O \) looks at two stars, rotating about the center of mass of the pair. Let be at rest relative to \( O \) (for simplicity), let the orbits of the stars be circular, and let lie on the normal through to the plane of the orbits. Then Eisner shows that the stars must appear to describe circular orbits of angular radii \( S_0 \), which he calculates explicitly. The result is devastating for the idea that aberration depends only on relative velocity source-observer. Indeed, if it were so, “the skies should be filled with binary stars of apparent separation of order forty seconds.” [13-5]

A further support for Ives' conclusion came from Hayden (1993), who remarked that some astronomical tables list five pages of binaries that can be seen in the Sagittarius alone with a small telescope. A spectroscopic binary, Mizar A, has well known orbital parameters, from which Hayden calculates an observable angular separation of 1” 10”.5 if aberration were due to relative velocity. The empirical value is less than 0.01”, clearly incompatible with the relativistic prediction [13-6].

The problem of aberration is solved by the equivalent transformations predicting that the \( v \) appearing in (13.1) is the Earth absolute velocity and that the aberration phenomenon is due to the variations of \( v \) generated by the orbital motion of our planet. Obviously in this case the aberration angle has to be the same for all the stars in the sky!

Next let us come to the proof of the aberration formula, eq. (13.1). The following notation will be used:

\( \vec{v} \): absolute velocity of Earth (variable during the year);
\( \vec{c} \): vector velocity of a light ray with respect to \( S_0 \). Given the isotropy of \( S_0 \) its modulus has the constant value \( c \).
\( \vec{c}_e \): velocity of the same light-ray relative to the Earth.

The transformations (7.2) from \( S_0 \) to the Earth frame \( S \) in two dimensions and in differential form are

\[
\begin{align*}
\frac{dx_E}{dt_E} &= \frac{(dx_0 - v dt_0)}{R} \\
\frac{dy_E}{dt_E} &= \frac{dy_0}{R} \\
\frac{dt_E}{dt_E} &= R dt_0 + c e (dx_0 - v dt_0) \\
\end{align*}
\]  
(13.2)
where the index $E$ denotes quantities calculated with respect to the inertial frame in which the Earth is instantaneously at rest. Consider an arbitrary pointlike object and define its velocities with respect to $S_0$ and $S$:

$$u_{Ex} = \frac{dx_E}{dt_E}; \quad u_{Ey} = \frac{dy_E}{dt_E}; \quad u_{0x} = \frac{dx_0}{dt_0}; \quad u_{0y} = \frac{dy_0}{dt_0}$$  \hspace{1cm} (13.3)

Dividing side by side the first two Eq.s (13.2) by the third one it follows

$$u_{Ex} = \frac{1}{R + e_1 (u_{0x} - \nu)} \cdot \left( u_{0x} - \nu \right) \quad ; \quad u_{Ey} = \frac{u_{0y}}{R + e_1 (u_{0x} - \nu)}$$  \hspace{1cm} (13.4)

Referring to the propagation of a light pulse and writing

\[
\begin{align*}
  u_{Ex} &= c_r \cos \theta \\
  u_{Ey} &= c_r \sin \theta \\
  u_{0x} &= c \cos \theta_0 \\
  u_{0y} &= c \sin \theta_0
\end{align*}
\]  \hspace{1cm} (13.5)

we get from (13.4)

\[
\begin{align*}
  c_r \cos \theta &= \frac{1}{R + e_1 (u_{0x} - \nu)} \cdot \left( u_{0x} - \nu \right) \\
  c_r \sin \theta &= \frac{c \sin \theta_0}{R + e_1 (u_{0x} - \nu)}
\end{align*}
\]  \hspace{1cm} (13.6)

Dividing side by side the two Eq.s (13.6) we get exactly the Eq. (13.1). Notice that the synchronization parameter $e_1$ disappears from the final result. All terms present in the right hand side of (13.1) ($\nu, c, \theta_0$) are measured in $S_0$ and are independent of clock synchronisation in $S$. They are the same in all equivalent theories, and the angle $\theta$ perceived on Earth (and its variations) are thus predicted to be exactly the same in all such theories. Even though all equivalent theories (TSR included) give the same mathematical predictions there are interpretative difficulties; for example in the meaning attributable to $\nu$. The TSR guided by relativism goes straight to the adoption of an impossible relative velocity interpretation, but the theory fully in agreement with the data is the one based on the inertial transformations and absolute simultaneity ($e_1 = 0$).


[13-3] [CM, p. 25].


Chapter 14

The differential retardation of clocks

In 1957 the Australian physicist G. Builder showed that the differential retardation effect for two clocks which separate to reunite later again can be considered with respect to an arbitrary inertial reference frame. Such an effect (read on the clocks) appears to be the same to observers in all states of motion, and in this sense is absolute (the word “invariant” is also in use). Builder concluded that the emergence of an absolute effect consequence of velocity implies the existence of a privileged inertial frame, in the sense that motion relative to this frame assumes an absolute significance and is associated with absolute effects. A relativistic theory is thus seen to contain also absolute features (“clock paradox”).

Builder’s paper is mostly qualitative and in some points difficult to understand. In this chapter I offer my hopefully clearer reformulation, based on two assumptions:
A1. The velocity of light relative to an inertial system, $S_0$, is $c$ in all directions, so that clocks are synchronized in $S_0$ with the Einstein method and one way velocities relative to $S_0$ can be measured;
A2. A clock moving with speed $u(t_0)$ relative to $S_0$ during the $S_0$ time interval $dt_0$ marks a proper time increase $d\tau$ given by:

$$d\tau = dt_0 \sqrt{1-u^2(t_0)/c^2}$$

The theory of special relativity (TSR) satisfies the above assumptions in all inertial systems. The theory of the “equivalent transformations” (TET) takes $S_0$ as the privileged system relative to which A1 and A2 are satisfied as well. Therefore the consequences deduced from A1 and A2 are valid both in the TSR and in the TET. But the principle of relativity, the metrics of Minkowski space and/or the gravitational potential of the fictitious forces can well be suspected to have nothing to do with the essence of the matter. Indeed in the text below we show that only velocities influence the rate of the periodic phenomenon on which the working of a clock is based. This leads to the necessity to distinguish gravitodynamic from gravitostatic effects.
14. The differential retardation of clocks

In the present section we discuss the differential retardation effect between separating and reuniting clocks ("clock paradox"). A variational method is used to show [14-1], both in the theory of relativity and in more general theories, that among all possible trajectories of a clock connecting two given points at two given times the rectilinear uniform motion requires the longest proper time. A complete resolution of the clock paradox is obtained by giving an exhaustive unified description of the possible situations. Absolute velocity (and nothing else) is thus seen to be responsible for the differential retardation effect. Hidden behind the relativism of Einstein's theory there must be a physically active background.


In the 1905 paper on the theory of relativity Einstein [14-2] presented the clock retardation prediction as follows: Imagine one of the clocks which mark the time $t_0$ when at rest relatively to the "stationary" inertial system $S_0$, and the time $t$ when at rest relatively to the "moving" inertial system $S$, to be located at the origin of the coordinates of $S$, where it marks the time $t$. What is it the rate of this clock, if viewed from the stationary system? The quantities $x_0$, $t_0$, and $t$, which refer to the position of the clock, satisfy: (i) $x_0 = vt_0$, and, (ii) the Lorentz transformation of time

$$ t = \frac{1}{R} \left( t_0 - \frac{v x_0}{c^2} \right) $$

where $R$ is given by Eq. (6.3). Therefore

$$ t = t_0 R = t_0 - (1 - R)t_0 $$

and one sees that the time $t$ marked by the clock is slow by $1 - R$ seconds per second with respect to the $S_0$ time $t_0$.

From this Einstein deduced another consequence, which has become famous as "clock paradox". If at the points $A$ and $B$ of $S_0$ there are two
synchronous stationary clocks; and if the clock at \(A\) is moved with the velocity \(v\) along the line \(AB\) to \(B\), then on its arrival at \(B\) the two clocks no longer show the same time, but the clock moved from \(A\) to \(B\) lags behind the other which has remained at \(B\) by \((1-R)t_0\), \(t_0\) being the duration of the journey from \(A\) to \(B\). Einstein considered evident that this result still holds good if the clock moves from \(A\) to \(B\) in any polygonal line, and also when the points \(A\) and \(B\) coincide. Assuming that the result obtained for a polygonal line holds also for a continuously curved line, he concluded: “If one of two synchronous clocks at \(A\) is moved on a closed curve with constant velocity until it returns to \(A\), the journey lasting \(t_0\) seconds, then by the clock which has remained at rest the travelled clock on its arrival at \(A\) will be \((1-R)t_0\) second slow.”

That is all. Clearly Einstein went beyond what he could say by considering only inertial systems, as he introduced also accelerated motions (in the vertices of the polygonal line and along the continuous curve). He did so in an intuitive way, without solid foundations, and yet he was closer to the correct result than with the 1918 paper (as we shall see). Today, the velocity dependent retardation of moving clocks and its independence of acceleration are well established empirical facts. A large amount of experimental evidence, in agreement with Einstein’s 1905 statements, points to the validity of the following clock retardation formula. We assume that if a clock \(U\), marking the time \(t_0\) when at rest in a certain isotropic inertial system \(S_0\), is set in motion with arbitrarily oriented and possibly variable velocity \(u(t_0)\) relative to \(S_0\), the rate of the time marked by \(U\) at \(S_0\) time \(t_0\) is given by

\[
d\tau = dt_0 \sqrt{1-u^2(t_0)/c^2}
\]

This \(\tau\) is exactly what an observer travelling with \(U\) finds by time reading on the clock itself. Therefore \(d\tau\) is the “proper time” variation of \(U\).

Clearly the idea behind (14.1) is that only the instantaneous velocity (and not the acceleration) fixes the rate of the clock. Let us check that this conclusion about \(d\tau\) is correct. In physics one can recognize the cause of a phenomenon by varying it and verifying the existence of corresponding variations of the effect. *Vice versa*, if arbitrary variations of a physical quantity \(Q\) do not modify the effect \(E\), one can exclude that \(Q\) is among the causes of
Let us apply this criterion to (14.1). If \( u(t_0) \) is varied, the square root factor is modified, and a corresponding variation of the proper time rate arises: therefore velocity can be claimed to be a cause of the proper time rate variation. On the contrary, if the acceleration is modified at time \( t_0 \) while \( u(t_0) \) remains the same, \( d\tau \) will not change. Therefore the acceleration has no effect on \( d\tau \) and cannot be counted among the causes of the variation of its rate.

As we saw in section 9 in a preliminary to the discussion of the Sagnac effect, all the ET lead to the validity of an equation identical to (14.1). Thus the discussion of the clock paradox superficially could appear to be the same for all equivalent theories. It is not really so, however, and a distinction shows up at least for the two most important theories, that is for:

- the TSR, built under a protective roof of relativism;
- the TIT, built for the search of the privileged frame.

As stressed by Prokhovnik [14-3] the resolution of the clock paradox in terms of absolute motion was found by the Australian physicist G. Builder [14-4] who showed that the differential retardation effect between two clocks which separate and reunite can be validly considered in respect to a single inertial reference frame. Such an effect (read on the clocks) appears obviously to be the same to observers in all states of motion, and in this sense it is absolute (the word “invariant” could also be used. I prefer “absolute” in the sense of being the opposite of “relative”). Builder concluded that the emergence of an absolute effect consequence of velocity implies the existence of a privileged inertial frame, in the sense that motion relative to this frame assumes an absolute significance and is associated with absolute effects.

14.2. A new proof of the differential retardation theorem

Builder’s paper is mostly qualitative and in some points difficult to understand. My hopefully clearer reformulation of his argument is based on two simple assumptions:

A1. The velocity of light relative to an inertial system, \( S_0 \), is "c" in all directions, so that clocks can be synchronized in \( S_0 \) with the Einstein method and one way velocities relative to \( S_0 \) can be measured;
A2. A clock moving with speed \( u(t_0) \) relative to \( S_0 \) during the \( S_0 \) time interval \( dt_0 \) marks a (proper) time increase \( d\tau \) given by (14.1).

The theory of special relativity (TSR) is well known to satisfy the above assumptions in all inertial systems. The theory of the “equivalent transformations” [14-5] accepts \( S_0 \) as the privileged system relative to which A1 and A2 are satisfied as well. Therefore all the consequences deduced from A1 and A2 are valid both in the TSR and in the theory of the equivalent transformations. But the principle of relativity, the metrics of Minkowski space and/or the gravitational potential of the fictitious forces do not seem to have anything to do with the essence of the matter. We will see that only velocities influence the rate of the periodic phenomenon on which the working of a clock is based.

In this section we consider only one spatial dimension for simplicity. All arguments can however be generalized to three dimensions rather easily [14-6]. According to A2 the proper time increase \( T \) marked by a clock moving from the point \( a \) at time \( t_{0a} \) to the point \( b \) at time \( t_{0b} \), both fixed in \( S_0 \), (see Fig. 16) is

\[
T = \int_{t_{0a}}^{t_{0b}} dt_0 \sqrt{1 - \frac{u^2(t_0)}{c^2}}
\]

(14.2)

where \( u(t_0) \) is the velocity, given by

\[
u(t_0) = \frac{dx_0}{dt_0}
\]

(14.3)

and \( x_0 = x_0(t_0) \) is the equation of motion of the clock on some “trajectory” in the \((x_0, t_0)\) plane connecting the points \( a, b \) of Fig. 16. We consider a second (“varied”) trajectory, very near to the original one, as follows

\[
x_0(t_0) \rightarrow x_0(t_0) + \delta x_0(t_0)
\]

(14.4)

with

\[
\delta x_0(t_{0a}) = \delta x_0(t_{0b}) = 0
\]

(14.5)
According to Eq. (14.5) the clock on the varied trajectory occupies the points \(a\) and \(b\) at the same times \(t_{0a}\) and \(t_{0b}\) as on the unvaried trajectory. This clearly corresponds to the situation in which two clocks separate at point \(a\) at time \(t_{0a}\), follow different trajectories and reunite again in point \(b\) at time \(t_{0b}\). In such a case both trajectories would be real and occupied by a clock. As a consequence of (14.4) also the velocity has a variation

\[
\begin{align*}
  u(t_0) & \rightarrow u(t_0) + \delta u(t_0) \\
  \text{with } u(t_0) \text{ given by (14.3) and }
\end{align*}
\]

\[
\delta u(t_0) = \frac{d}{dt_0} \delta x_0(t_0)
\]

Figure 16. A space and time diagram showing a “trajectory” between two points, and a second varied trajectory between the same points.

The proper time integral will correspondingly become
\[ T + \delta T = \int_{t_0}^{t_0'} dt_0 \sqrt{1 - [u(t_0) + \delta u(t_0)]^2 / c^2} \] (14.8)

From (14.7) and (14.8) it follows, for small variations

\[ \delta T = -\frac{1}{c^2} \int_{t_0}^{t_0'} dt_0 \frac{u(t_0)}{\sqrt{1 - u(t_0)^2 / c^2}} \frac{d}{dt_0} \delta x_0(t_0) \] (14.9)

Integrating by parts one gets

\[ \delta T = -\frac{1}{c^2} \left[ \frac{u(t_0) \delta x_0(t_0)}{\sqrt{1 - u(t_0)^2 / c^2}} \right]_{t_0}^{t_0'} - \int_{t_0}^{t_0'} dt_0 \frac{d}{dt_0} \left[ \frac{u(t_0)}{\sqrt{1 - u(t_0)^2 / c^2}} \right] \delta x_0(t_0) \] (14.10)
Due to (14.5) the first term in the right hand side vanishes. The derivative in
the second term gives

$$\delta T = \frac{1}{c^2} \int_{t_0a}^{t_0b} dt_0 \left[ \frac{\dot{u}(t_0)}{1-u(t_0)^2/c^2} \right]^{3/2} \delta x_0(t_0)$$

(14.11)

where $\dot{u}(t_0) = du(x_0)/dt_0$. Clearly, $\delta T = 0$ for arbitrary $\delta x_0(t_0)$ satisfying
(14.5) if and only if $\dot{u}(t_0) = 0$ at all times. This is like saying that the
extremum proper time $T$ of all motions is the motion with constant velocity

$$u_1 = \frac{x_{0b} - x_{0a}}{t_{0b} - t_{0a}}$$

(14.12)
for which the proper time integral (14.2) takes the value

\[ T_1 = (t_{0b} - t_{0a}) \sqrt{1 - u_1^2 / c^2} \]  

(14.13)

Among all motions connecting \( a \) and \( b \) this extremum is unique, as it can be obtained for \( \dot{u}(t_0) = 0 \) only. Therefore it gives either the maximum or the minimum proper time of all possible motions from \( a \) to \( b \), not only of those obtained with an infinitesimal deformation of the straight line.

That the extremum is actually a maximum can be seen as follows. One has

\[ \sqrt{1 - (u_1 + \delta u)^2 / c^2} \equiv \sqrt{1 - u_1^2 / c^2} - \frac{1}{c^2} \frac{u_1 \delta u}{\sqrt{1 - u_1^2 / c^2}} - \frac{1}{2c^2} \frac{(\delta u)^2}{(1 - u_1^2 / c^2)^{3/2}} \]

(14.14)

But \( u_1 \) is constant and \( \delta u \) satisfies (14.7) so that, after integration, the first order variation of \( T_1 \) arising from (14.14) vanishes due to (14.5). The second order variation

\[ \delta^2 T = \frac{1}{2c^2} \int_{t_{0a}}^{t_{0b}} dt_0 \frac{[\delta u(t_0)]^2}{[1-u_1^2 / c^2]^{3/2}} \]

(14.15)

is clearly negative for all possible velocity variations. Therefore, moving away from the constant velocity line of Fig. 17 to a different line connecting \( a \) and \( b \) implies in all cases a decrease of the elapsed proper time. Then the found extremum is a maximum. Comparing the motion with velocity \( u_1 \) with any different motion from \( a \) to \( b \) with velocity \( u_2(t_0) \) one has

\[ \Delta T = T_1 - T_2 = \int_{t_{0a}}^{t_{0b}} dt_0 \left[ \sqrt{1-u_1^2 / c^2} - \sqrt{1-u_2(t_0)^2 / c^2} \right] > 0 \]

(14.16)
As a difference of proper times, $\Delta T$ is exactly what two observers who traveled with the clocks find by direct comparison of the clocks readings.

The clock moving with rectilinear uniform motion can be considered at rest in a different inertial system. Therefore the previous argument can be taken to describe the typical “clock paradox” situation from the point of view of the particular inertial system $S_0$ we have considered. We should recall that we also found that the choice $e_1 = 0$ is the only one allowing for a treatment of accelerations rationally connected with the physics of inertial systems $S_0$: see chapters 9, 10 and 11 of this book.

Assuming the inertial transformations, the $S_0$ system is initially considered privileged, and the velocity of light relative to it isotropic. Other inertial systems are described as "moving" and relative to them the observers detect an anisotropic velocity of light. One can add that from the point of view of the inertial transformations the validity of the relativity principle appears accidental, more than fundamental.

14.3. The differential retardation according to General Relativity.

The 1905 formulation of the clock paradox had an implication that probably Einstein did not like. The delay is an absolute effect, as all observers agree that the clock moving with variable velocity marks a smaller time. They disagree, however, on the numerical value of this variable velocity at any instant of proper time. In relativity all potential observers (forming an infinite set) are completely equivalent, so that, in a sense, one can say that the clock velocity assumes at any time all conceivable values. But a quantity having at the same time infinitely many values is totally undefined. In this way the presumed cause of the differential retardation would seem to vanish into nothingness. But, no, this is not physically reasonable, obviously the cause of a real physical effect should be concrete as well, in spite of the evasive description coming from the theory. Therefore causality implies that velocity itself should be well defined, that is, relative to a physically active reference background (ether) that defines at the same time the privileged inertial reference frame.

It is no surprise, then, that to escape from conclusions favourable to the ether the original formulation was completed with a later one based on the theory of general relativity (TGR) [14-7], whose essential points we will now review. Let $S$ be an inertial reference system. Further, let $U_1$ and $U_2$ be two exactly similar clocks working at the same rate when at rest relatively to $S$. If one of the clocks - let us say $U_2$ - is in a state of uniform translatory motion
relative to $S$, then, according to the TSR it works more slowly than $U_1$, which is at rest in $S$.

At this point Einstein adds an interesting remark: “This result seems odd in itself. It gives rise to serious doubts when one imagines the following thought experiment.” In the thought experiment $A$ is the origin of $S$, and $B$ a different point of the positive $x$-axis. The two clocks are initially at rest at $A$, so that they work at the same rate and their readings are the same. Next, a constant velocity in the direction $+x$ is imparted to $U_2$, so that it moves towards $B$. At $B$ the velocity is reversed, so that $U_2$ returns towards $A$. When it arrives at $A$ its motion is stopped, so that it is again at rest near $U_1$. Since $U_2$ works more slowly than $U_1$ during its motion along the line $A\ B$, $U_2$ must be behind $U_1$ on its return.

Now comes the problem, says Einstein. According to the principle of relativity the whole process must surely take place in exactly the same way if it is considered in a reference frame $S'$ sharing the movement of $U_2$. Relatively to $S'$ it is $U_1$ that executes the to-and-fro movement while $U_2$ remains at rest throughout. From this it would seem to follow that, at the end of the process, $U_1$ must be behind $U_2$, which contradicts the former conclusion.

But, Einstein adds, the TSR is inapplicable to the second case, as it deals only with inertial reference frames, while $S$ is at times accelerated. Only the TGR deals with accelerated frames. From the point of view of the TGR, one can use the coordinate system $S'$ just as well as $S$. But in describing the whole process, $S$ and $S'$ are not equivalent as the following comparison of the movements shows.

<table>
<thead>
<tr>
<th>Description relative to the $S$ Reference System</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The clock $U_2$ is accelerated by an external force in the direction $+x$ until it reaches the velocity $\nu$. $U_1$ is at rest in $S$, now as in all the subsequent steps.</td>
</tr>
<tr>
<td>2. $U_2$ moves with constant velocity $\nu$ to the point $B$ on the $+x$-axis.</td>
</tr>
<tr>
<td>3. $U_2$ is accelerated by an external force in the direction $-x$ until it reaches the velocity $\nu$ in the direction $-x$.</td>
</tr>
<tr>
<td>4. $U_2$ moves with constant velocity $\nu$ in the direction $-x$ back to the neighbourhood of $U_1$.</td>
</tr>
</tbody>
</table>
5. $U_2$ is brought to rest very near to $U_1$ by an external force.

**Description relative to the $S'$ Reference System**

1. A gravitational field, oriented along $-x$, appears, in which the clock $U_1$ falls with an accelerated motion until it reaches the velocity $v$. When $U_1$ has reached the velocity $v$, the gravitational field vanishes. An external force applied to $U_2$ prevents $U_2$ from being moved by the gravitational field. Thus $U_2$ remains at rest in $S'$.
2. $U_1$ moves with constant velocity $v$ to a point $B'$ on the $-x$-axis. $U_2$ remains at rest in $S'$.
3. A homogeneous gravitational field in the direction $+x$ appears, under the influence of which $U_1$ is accelerated in the direction $+x$ until it reaches the velocity $v$, whereupon the gravitational field vanishes. $U_2$ is kept at rest in $S'$ by an external force.
4. $U_1$ moves with constant velocity $v$ in the direction $+x$ into the neighbourhood of $U_2$. $U_2$ remains at rest in $S'$.
5. A gravitational field in the direction $-x$ appears, which brings $U_1$ to rest. The gravitational field then vanishes. $U_2$ is kept at rest in $S'$ by an external force.

The second description, based on the principle of equivalence between fictitious and gravitational forces, has been introduced essentially to protect the theory from the loss of relativism. According to both descriptions, at the end of the process the clock $U_2$ is retarded by a definite amount compared with $U_1$. With reference to $S'$ this is explained by noticing that during the stages 2 and 4, the clock $U_1$, moving with velocity $v$, works more slowly than $U_2$, which is at rest. But this retardation is overcome by the quicker working of $U_1$ during stage 3. For, according to the GTR, a clock works the faster the higher the gravitational potential in the point where it is placed, and during stage 3 $U_1$ is indeed in a region of higher gravitational potential than $U_2$. A calculation made with instantaneous acceleration shows that the consequent advancement amounts to exactly twice as much as the retardation during stages 2 and 4 [14-7]. Arrived at this conclusion Einstein states: “This completely clears up the paradox.”
We said that a clock works faster the larger is the gravitational potential $\phi$ in the region in which it is placed. This prediction of the TGR is confirmed by the experiments performed in the gravitational field of the Earth, so that at first sight the 1918 reasoning could seem to be a consequence of the empirical facts. The mathematical treatment of the clock paradox situation given by the TGR is thought to lead to the right result by describing the retardation of $U_2$ as a consequence of $\phi$ [14-7]. Yet the theory shows its weakness in a different way.

14.4. Argument against the general relativistic treatment
The differential retardation becomes more difficult to describe in reasonable physical terms in the framework of the GTR if two stationary clocks in different fixed positions of the same inertial reference system are considered. The accelerated part of the trip of a third clock, even if very far away, modifies the reciprocal synchronization of these stationary clocks if one does not give up the principle of equivalence idea that the gravitational potential of the fictitious forces has an influence on the time marked by the stationary clocks.

Three clocks are given, $U_1, U_2$ and $U$, which mark the time in the same way if placed at rest with respect to one another. The first two are constantly at rest on the $x$ axis of the inertial reference system $S$, in the points $P_1$ and $P_2$ with respective coordinates $x_1$ and $x_2$ ($x_2 > x_1 > 0$). The third clock, $U$, moves on the same axis in the $+x$ direction with initially constant velocity $V$. It is set on the time $\tau = 0$ when it passes on the origin $O$ of the coordinates of the system $S$, whose observers also adopt the time $t = 0$ when $U$ passes on $O$. 

Figure 18. The three clock experiment.
The TSR predicts that $U_2$ and $U$, meeting for the first time in $p_i$, mark respectively the times

$$t_1 = \frac{x_1}{V}, \quad \tau_1 = \frac{x_1}{V} \sqrt{1 - V^2/c^2} \tag{14.17}$$

Thus $U$, reaching the point $p_i$, has accumulated on $U_1$ the delay

$$T(OP_1) \equiv t_1 - \tau_1 = \frac{x_1}{V} \left[1 - \sqrt{1 - V^2/c^2}\right] \tag{14.18}$$

Similarly the TSR predicts that $U_2$ and $U$, meeting for the first time in $P_2$, mark respectively the times

$$t_2 = \frac{x_2}{V}, \quad \tau_2 = \frac{x_2}{V} \sqrt{1 - V^2/c^2} \tag{14.19}$$

Thus $U$, reaching the point $P_2$, has accumulated on $U_2$ the delay

$$T(OP_2) \equiv t_2 - \tau_2 = \frac{x_2}{V} \left[1 - \sqrt{1 - V^2/c^2}\right] \tag{14.20}$$

When $U$, continuing in its motion, reaches the point $P$ with coordinate $L$ (with $L > x_2 > x_1$) $U_1$ and $U_2$ mark the time $t$ while $U$ marks the time $\tau$ given by

$$t = \frac{L}{V}, \quad \tau = \frac{L}{V} \sqrt{1 - V^2/c^2} \tag{14.21}$$

Reaching the point $P$ $U$ has accumulated on $U_1$ and $U_2$ the delay

$$T(OP) \equiv t - \tau = \frac{L}{V} \left[1 - \sqrt{1 - V^2/c^2}\right] \tag{14.22}$$
Clearly the delays accumulated by \( U \) in the segments \( P_1P \) and \( P_2P \) are

\[
\begin{align*}
T(P_1P) &= T(OP) - T(OP_1) = \frac{L - x_1}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right] \\
T(P_2P) &= T(OP) - T(OP_2) = \frac{L - x_2}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right]
\end{align*}
\]

(14.23)

respectively.

Now \( U \), having reached the point of coordinate \( L \), suddenly inverts its motion passing instantaneously from velocity \( V \) to \( -V \). Symmetry implies that when \( U \) meets \( U_1 \) and \( U_2 \) for the second time the delays given by (14.23) will be doubled, that is (with obvious meanings of the new symbols):

\[
\begin{align*}
T(P_1PP_1) &= 2 \frac{L - x_1}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right] \\
T(P_2PP_2) &= 2 \frac{L - x_2}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right]
\end{align*}
\]

(14.24)

These delays, just as the clock readings from which they are deduced [see (14.17), (14.19) and (14.21)], are invariant, that is they are the same for all observers (whether inertial or not) as they correspond to objective events. In fact at the time at which two clocks overlap all observers, independently of their state of motion, must necessarily read the same time on the display of each of them.

Let us discuss the same phenomenon in a noninertial reference frame from the point of view of the general theory of relativity. Of our three clocks, \( U_1, U_2 \) and \( U \), the third one is at rest in a non inertial reference system \( \mathcal{S} \), while the first two perform with respect to the third one a to and fro movement which is only a different way to perceive the motions described above.

The TGR describes the effects of the paths covered by \( U_1 \) and \( U_2 \) with constant velocity exactly as the TSR would, obtaining naturally conclusions equal and opposite to those given by (14.24), deduced in \( \mathcal{S} \), by stating that \( U_1 \) and \( U_2 \) have a retardation with respect to \( U \), so that the “retardation” accumulated by \( U \) is negative (actually it is an advancement) and given by
\[
\begin{align*}
\tilde{T}_{vel}(P_1PP_1) &= -2L - \frac{x_1}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right] \\
\tilde{T}_{vel}(P_2PP_2) &= -2L - \frac{x_2}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right]
\end{align*}
\]

(14.25)

where the tilde indicates that the calculation is performed in \( \tilde{S} \). According to the TGR one must also consider the equivalence between inertial and gravitational forces and add the effects of the potential \( \phi(x) \) of the vectorially constant gravitational field perceived in \( \tilde{S} \) at the time of acceleration. The constancy of the field ensures that \( \phi(x) \) depends linearly on the distance from \( U \). Well, it was shown by Einstein [14-7], Møller [14-8] and Iorio [14-9] that such effects manifest themselves on \( U_1 \) and \( U_2 \) having in all cases opposite sign and double value of the effects due to velocity as given by (14.25). That is

\[
\begin{align*}
\tilde{T}_\phi(P_1PP_1) &= 4L - \frac{x_1}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right] \\
\tilde{T}_\phi(P_2PP_2) &= 4L - \frac{x_2}{V} \left[ 1 - \sqrt{1 - \frac{V^2}{c^2}} \right]
\end{align*}
\]

(14.26)

Naturally the total retardations obtained in \( \tilde{S} \) are

\[
\begin{align*}
\tilde{T}(P_1PP_1) &= \tilde{T}_{vel}(P_1PP_1) + \tilde{T}_\phi(P_1PP_1) \\
\tilde{T}(P_2PP_2) &= \tilde{T}_{vel}(P_2PP_2) + \tilde{T}_\phi(P_2PP_2)
\end{align*}
\]

(14.27)

so that one gets the expected result

\[
\begin{align*}
\tilde{T}(P_1PP_1) &= T(P_1PP_1) \\
\tilde{T}(P_2PP_2) &= T(P_2PP_2)
\end{align*}
\]

(14.28)

as it is necessary given the invariant nature of \( T(P_1PP_1) \) and \( T(P_2PP_2) \).

In conclusion we can say that also the TGR leads to the right prediction of the differential retardation of two clocks that separate to reunite later again. This treatment however contains also disputable features.
The TSR and the TGR agree on the obvious idea that clocks at rest in the same inertial reference system, such as \( U_1 \) and \( U_2 \), maintain in time their synchronization, at least if there are no other clocks accelerating, such as \( U \) in our previous discussion. In other words the retardations due to velocity (14.25) are physiological and do not present disputable features, given that the whole TSR is based on the implicit idea that uniform motions of clocks do not modify a preestablished synchronization of other clocks. Naturally the numerical difference of \( \tilde{T}_{vel}(P_1PP_1) \) and \( \tilde{T}_{vel}(P_2PP_2) \) is easily explained by the fact that they arise from paths of different length and duration of the respective clocks.

Much more puzzling is the case of the retardations (14.28), due to the potential, which arise from the same very short time of acceleration of \( U \). Their different value is due to the fact that \( \phi(x_1) \neq \phi(x_2) \), but it implies a desynchronization between \( U_1 \) and \( U_2 \). The situation is similar to the observable change of the rhythm of clocks in the gravitational field of the Earth accompanying a modification of their height over sea level. If \( U_1 \) and \( U_2 \) had been synchronized in such a way as to produce a speed of light equal to \( c \) from \( P_1 \) to \( P_2 \) before \( U \) accelerated, they cannot do so anymore after acceleration, given that to their marked times the invariants \( \tilde{T}_{\phi}(P_1PP_1) \) and \( \tilde{T}_{\phi}(P_2PP_2) \) were added. In other words, if before acceleration of \( U \) a flash of light left \( P_1 \) at time \( t_1 \) (marked by \( U_1 \)) and reached \( P_2 \) at time \( t_2 \) (marked by \( U_2 \)), one had

\[
\frac{t_2 - t_1}{x_2 - x_1} = \frac{1}{c}
\]

(14.29)

After the acceleration of \( U \) one should instead observe a fictitious speed of light \( \tilde{c} \) given by

\[
\frac{1}{\tilde{c}} = \frac{[t_2 + \tilde{T}_{\phi}(P_2PP_2)] - [t_1 + \tilde{T}_{\phi}(P_1PP_1)]}{x_2 - x_1}
\]

(14.30)

From (14.30), using (14.29) and (14.26) one gets
\[
\frac{1}{\tilde{c}} = \frac{1}{c} + 4 \frac{1 - \sqrt{1 - V^2/c^2}}{c} \frac{V}{c}
\]

(14.31)

showing that the effect is certainly non negligible. For example, for small values of \(V/c\) one has

\[
\tilde{c} \equiv c \left(1 - 2 \frac{V}{c}\right)
\]

(14.32)

It is possible to check experimentally this prediction, even if it is very difficult to believe that something bound to the acceleration of a very far away clock can really modify the time marked by other clocks motionless in an inertial frame. It is easier to believe that the potential of the inertial forces \(\Phi(x)\) has in reality no possibility to modify the time marked by other clocks. After all, according to the equivalence principle, \(\Phi(x)\) does not describe a gravitostatic field like that of the Earth, but arises only from accelerations in non inertial reference systems.

This conclusion had been anticipated by Builder. In order to save the approach of the TGR one could perhaps assume that the action of \(\Phi(x)\) on the clocks \(U_1\) and \(U_2\) exists exclusively from the subjective point of view of an observer at rest with respect to \(U\). The point is that such an observer should assume different effects on \(U_1\) and \(U_2\), an idea easily falsified by observers bound to \(U_1\) and \(U_2\) who had kept measuring the speed of light from \(P_1\) to \(P_2\) during the whole trip of \(U\).

We can conclude that the gravitational potential of the fictitious forces exerts no action on the clocks, contrary to Einstein’s 1918 opinion. The gravitational fields in the accelerated systems are not ordinary static fields like that of the Earth, but arise from the accelerations of bodies [14-10]. Einstein assumed that these fields had on clocks the same action as ordinary fields, but we can now say that on this particular point he was not right, in spite of the very probable correctness of the general idea of equivalence between fictitious and gravitational forces. One finds an analogy in the magnetic field, which can be considered a dynamical manifestation of the electric field, but has quite different interaction properties.

As we can see, Einstein rejected the existence of a stationary ether violating the relativity principle. In his idea the new ether could be introduced...
instead: it would respect the relativity principle because it would not be a medium with its own state of motion. The state of the new ether (its metric behaviour) would be described by tensor $g_{\mu \nu}$, and that is why Einstein wrote "state $g_{\mu \nu} = \text{ether}$".

Finally, one could ask, who won in the Einstein-Lorentz debate? In my opinion nobody did and the confrontation ended with a tie. None of the two champions of scientific rationality admitted having made a mistake, both ended the debate more convinced than ever of owning the potentiality for great discoveries. The conclusion is typical of the confrontations between high level physicists, divided only by different philosophical outlooks. Other examples are Newton vs. Leibnitz on the nature of space and time and Einstein vs. Bohr on the foundations of quantum theory.

The lack of a logical-scientific victory of Einstein over Lorentz is another reason why today the realistic outlook of the Dutch physicist is still an open possibility for the development of physics. Collecting evidence in favor of our thesis of superiority of the inertial transformations is not too difficult, as we pointed out in chapter 7. It would be enough to build a muon storage ring of a different size, but transporting muons with the same velocity as the original ring, to have an example of indifference of the muon decay on the variations of the gravitational potential. But probably not even that is needed: given the muon velocity the decay rate is the same for a rectilinear beam (zero gravitational potential) and for a storage ring beam (large gravitational potential.)
[14-8] [CM, § 98].
Chapter 15

The Lorentz ether

Lorentz worked on a line of thought favourable to ether, arguing that of all frames of reference, the one should be preferred in which ether is at rest. Clocks at rest in this frame show the real physical time, and simultaneity, as shown by them, is not relative but holds true objectively. Lorentz saw the electromagnetic field of the ether as mediator of interactions between charged objects, and changes in this field could propagate with a speed not higher than the speed of light. Lorentz suggested that not only the electrostatic fields, but also the molecular forces (whatever they may be) are affected in such a way that the size of a body along the line of motion is reduced according to \( \ell = \ell_0 \sqrt{1-\frac{v^2}{c^2}} \) where \( \ell_0(\ell) \) is the distance separating two points \( A \) and \( B \) of the body, the line \( AB \) being parallel to the body velocity, as measured in the frame \( S_0 \) (resp. \( S \)).

Lorentz conceived rather asymmetrically the contraction of moving bodies (“a real physical phenomenon”) and the retardation of moving clocks (“only a useful convention”). New hypotheses allowed him to develop a theory formally equivalent to special relativity, but based on the existence of ether. The conceptual differences are very important as in Lorentz’s theory the ether has a fundamental role, while the relativity principle seems to be only coincidental: it is not in the set of the assumptions and becomes a kind of qualitative theorem.

Given that it was impossible to distinguish experimentally Lorentz’s formulation from Einstein’s, the conceptual differences seemed to be metaphysical. In principle every physicist could have chosen the formulation he liked most; in practice the great diffusion of negative ideologies in the European culture of those times strongly favoured the acceptance of Einstein’s relativism. In spite of the general trend Einstein’s works failed to convince Lorentz that the ether was superfluous. Lorentz defended the concept of the ether until the end of his life. When Lorentz received the final formulation of the General Theory of Relativity he still believed that Einstein's theory could be reconciled with an ether at rest. He tried to convince Einstein, by writing to him a long letter (1916), which gave rise to a friendly but firm confrontation between two giants of the scientific thought on the foundations of modern physics.

One could ask, who won in the Einstein-Lorentz debate? In my opinion nobody did and the confrontation ended with a tie. None of the two champions of scientific rationality admitted having made a major mistake, both ended the debate convinced of owning the potentiality for great discoveries. The conclusion is typical of the confrontations between high level physicists, divided by different philosophical
outlooks. Other examples are Newton vs. Leibnitz on the nature of space and time and Einstein vs. Bohr on the foundations of quantum theory.

15. The Lorentz ether

The aberration of the starlight was explained by Bradley in terms of the corpuscular theory, but it was Young (1804) who first showed how to explain the phenomenon on the wave hypothesis. He commented: "Upon considering the phenomenon of the aberration of the stars, I am disposed to believe that the luminiferous aether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees". [15-1]

In 1908 J.J. Thomson gave a nice argument in favour of the existence of ether. He argued that when we have a system whose momentum does not remain constant, the conclusion we should draw is not that Newton’s third law fails, but that our system, instead of being isolated as we had supposed, is connected with another system which can store up the momentum lost by the primary. In this way the motion of the complete system can be in agreement with the ordinary laws of dynamics. In the case of electrified bodies one understands that they must be connected with some invisible universe, which can be called the ether, and that this ether must possess mass and be set in motion when the electrified bodies are moved. Physicists are thus surrounded by an invisible universe with which they can get into touch by means of electrified bodies. [15-2]

Initially the theory of relativity had convinced only very few people that ether had to be eliminated. We said already that Einstein himself changed his opinion after 1920, probably under Lorentz’s influence, besides the impact of his own formulation of the equivalence principle. Furthermore Poincaré continued to write about ether, for example in a 1912 conference to the French Physical Society entitled “Les rapports de la matière et de l’éther” [15-3].

Poincaré, however, did not seriously believe in the reality of ether, if in 1889 he could write: “Whether the ether exists or not matters little - let us leave that to the metaphysicians; what is essential for us is that everything happens as if it existed, and that this hypothesis is found to be suitable for the explanation of phenomena. After all, have we any other reason for believing in the existence of material objects? That, too, is only a convenient hypothesis; only, it will never cease to be so, while some day, no doubt, the ether will be
thrown aside as useless.” [15-4] Of course Poincaré is well known for his conventionalism. He remarked that astronomers like Römer in measuring the speed of light, simply assume that it is a constant (in time) and that it is the same in all directions. Without this assumption it would not be possible to infer the speed of light from astronomical observations, as Römer did by using the occultations of the moons of Jupiter.

Poincaré was the first (1904) to give the modern formulation of the relativity principle [15-5] (with his words: The principle of relativity, according to which the laws of physical phenomena must be the same for a stationary observer as for one carried along in a uniform motion of translation, so that we have no means, and can have none, of determining whether or not we are being carried along in such a motion.) Now, the relativity principle and the ether were, so to say, competitors in the attempts at explaining the experimental situation regarding the propagation of light, and every positive development of the one was inevitably a blow to the other. So that the role of Poincaré, discoverer of the modern relativity principle, appears once more as philosophically ambiguous.

As we know a similar ambiguity was implicit in Einstein’s views, so much that in the first section we used the title “Einstein positivist/realist”. This did not stop him from making great discoveries. In fact, the father of relativity was the first man on earth to conclude that all moving clocks must slow down according to the rule:

$$\Delta t = \Delta t_0 \sqrt{1 - \frac{v^2}{c^2}}$$

(15.1)

where $\Delta t$ is the time elapsed between any two events as measured in the rest frame $S$ of the clock; $\Delta t_0$ is the time elapsed between the same two events as measured in the stationary frame $S_0$; $v$ is the clock velocity, then also the relative velocity of $S$ and $S_0$. This was one of the many successful consequences of the theory of special relativity.

Concerning the vacuum, Einstein's point of view in 1905 was more or less the following: ether does not exist, therefore it does not make any sense to consider motion with respect to nothing. It makes sense only to describe it with respect to concrete objects. The contraction of rods and the slowdown of clocks are always relative to observers who see them in motion, and a perfect symmetry exists (physical and philosophical) between the conclusions of different inertial observers. Considering a clock in uniform motion relative to
the different inertial observers $O_1$, $O_2$, ... $O_n$, who see it moving with respective velocities $v_1$, $v_2$, ... $v_n$, its time keeping turns out to be retarded by the usual velocity dependent square root factors.

A legitimate question seems to be: “What happens really to the clock, which time is it really marking?” The relativistic answer is that the question itself does not make sense, because all the points of view of the different observers are equally valid. In this way the philosophy of relativism and subjectivism takes control, in physics, for the observations of the inertial observers.

Paul Ehrenfest felt strongly the existence of this kind of problems. The special theory of relativity based on the negation of ether requires a complete equivalence of the inertial observers, since there is no reason why they should be inequivalent, given that they move with respect to nothing. Nevertheless, if one adopts the equivalence principle, that Einstein formulated in 1916, inertia has its origin in the gravitational effects of distant masses, effects mediated by physical fields present in empty space. But the word “ether” and the word “field” mean more or less the same thing, a vacuum endowed with physical properties. This contradiction worried Ehrenfest who in 1919 wrote to Einstein: “Now one can no longer say that they move with respect to nothing, for now they move with respect to an enormous something! ... Einstein, my upset stomach hates your theory - it almost hates you yourself! How am I to provide for my students? What am I to answer to the philosophers?!!” [15-6]

One can certainly agree with Ehrenfest that the slowing down of the pace of moving clocks is completely incomprehensible if there is no concrete medium generating it. Some textbooks try to describe the Lorentz contraction as a purely apparent but essentially unreal phenomenon – similar to the apparent length of a train, which depends on distance, but the idea rises more questions than it answers.

Lorentz continued to work on a line of thought very favourable to ether, arguing that of all frames of reference, the one should be preferred in which ether is at rest. Clocks at rest in this frame show the real physical time, and simultaneity, as shown by them, is not relative but holds true in nature. The conditions of the ether are given by the electric field $E$ and by the magnetic field $H$: they represent the “states” of the ether. Fundamental for the ether model of Lorentz had been the ether theory of Fresnel, Maxwell’s equations and the electron theory of Clausius. Contrarily to the last named physicist, who accepted action at a distance between electrons, for Lorentz the
The electromagnetic field of the ether is the mediator of interactions, and changes in this field can propagate with a speed not higher than the speed of light. In 1889 Oliver Heaviside deduced from the Maxwell equations that the electrostatic field around a moving sphere is contracted along the line of motion by a factor $\sqrt{1 - v^2 / c^2}$. In the same year FitzGerald and three years later Lorentz suggested that not only the electrostatic fields, but also the molecular forces are affected in such a way that the size of a body along the line of motion is reduced according to

$$\ell = \ell_0 \sqrt{1 - v^2 / c^2}$$  (15.2)

where $\ell_0(\ell)$ is the distance separating two points $A$ and $B$ of the body, the line $AB$ being parallel to the body velocity, as measured in the frame $S_0(S)$. When FitzGerald proposed that moving objects contract in the direction parallel to velocity (1889), he attributed the phenomenon to the action of the ether. The story is told in a brilliant paper by John Bell: “We know that electric forces are affected by the motion of the electrified bodies relative to the ether and it seems a not improbable supposition that the molecular forces are affected by the motion and that the size of the body alters consequently.”

Concerning the Lorentz-FitzGerald contraction hypothesis, Lorentz wrote: “Surprising as this hypothesis may appear at first sight, yet we shall have to admit that it is by no means farfetched, as soon as we assume that molecular forces are also transmitted through the ether, like the electric and the magnetic forces of which we are able at the present time to make the assumption definitely. If they are so transmitted, the translation will very probably affect their action between two molecules or atoms, in a manner resembling the attraction or repulsion between charged particles”.

A modern epistemologist, Elie Zahar, supports the Lorentz conclusion by stating that Lorentz derived the Lorentz-FitzGerald contraction hypothesis from the molecular forces hypothesis which is in all senses non ad hoc. Furthermore the Michelson-Morley experiment, according to him, has lent strong support to the molecular forces hypothesis which conforms to the heuristic of the ether programme.

In 1916 Lorentz published the final edition of a book in which his different formulation of the relativistic theory was developed. Once more the idea was to start from some important but particular implications of
classical physics and to assume their general validity. Lorentz made three assumptions:

1. A rod in motion with respect to the ether with a velocity $v$ parallel to its length becomes shortened by a factor $\sqrt{1 - v^2 / c^2}$.

2. A clock in motion with respect to the ether with velocity $v$ has the time measuring periodic process slowed down by a factor $\sqrt{1 - v^2 / c^2}$.

3. Einstein’s convention for synchronising clocks is valid, that is the velocity of light can be assumed equal to $c$ in all directions and in all inertial reference frames.

One should also remember that Lorentz conceived rather asymmetrically the contraction of moving bodies (which he considered to be a real physical phenomenon) and the slowing down of moving clocks (which he considered to be only a useful convention). The time of his transformations was called by him “local time”, but only “for the sake of facilitating our mode of expression”.

The three previous hypotheses allowed him to develop a theory formally equivalent to special relativity, but based on the assumed existence of ether: the philosophical difference was thus important, even if the privileged frame, assumed to exist from the start, lost all peculiarity in the theoretical formalism by hiding, so to say, in the ensemble of all other inertial reference frames. Here is how Lorentz described the situation: “His [Einstein’s] results concerning electromagnetic and optical phenomena ... agree in the main with those which we have obtained in the preceding pages, the chief difference being that Einstein simply postulates what we have deduced, with some difficulty and not altogether satisfactorily, from the fundamental equations of the electromagnetic field. By doing so, he may certainly take credit for making us see in the negative result of experiments like those of Michelson, Rayleigh and Brace, not a fortuitus compensation of opposing effects, but the manifestation of a general and fundamental principle. Yet, I think, something may also be claimed in favour of the form in which I have presented the theory. I cannot but regard the ether, which can be the seat of an electromagnetic field with its energy and its vibrations, as endowed with a certain degree of substantiality, however different it may be from all ordinary
matter. In this line of thought, it seems natural not to assume at starting that it
can never make any difference whether a body moves through the ether or not,
and to measure distances and lengths of time by means of rods and clocks
having a fixed position relatively to the ether.” [15-14]

The theory of ether and electrons was developed mostly by Lorentz and
Poincaré in the years 1892 to 1906 and was based on the ether theory of
Fresnel. With his theory Lorentz could explain the aberration of light, the
Doppler effect and the Fresnel drag coefficient. He also succeeded in
explaining the Zeeman effect, theory for which he received the Nobel prize in
physics in 1902.

As stated above, the last edition (1916) of Lorentz’s “Theory of
electrons“[TE2] was the final point in the development of a classical ether
theory. Today this theory is often considered as some sort of “Lorentzian“ or
“neo-Lorentzian“ interpretation of special relativity. Even though the effects of
length contraction and time dilation were initially introduced relatively to a
privileged frame only (see the assumptions 1. and 2. above), the Lorentz
transformations were nevertheless deduced in later developments from the
postulates of the theory for any pair of reference frames. It is intuitive that two
theories sharing the Lorentz transformations are essentially the same thing. In
fact, it is not possible to distinguish experimentally Lorentz’s theory and
Special Relativity. The conceptual differences are very important, however, as
in Lorentz’s theory the ether has a fundamental role, while the relativity
principle seems to be only coincidental: it was not in the set of the
assumptions, as we saw, but it could be deduced as a sort of qualitative
theorem.

Given that it was impossible to distinguish experimentally Lorentz's
formulation from that of Einstein, the conceptual differences seemed to be
metaphysical. In principle every physicist could have chosen the formulation
he liked most; in practice the great diffusion of negative ideologies in the
European culture of the twenties and thirties strongly favoured the acceptance
of Einstein’s relativism. On the other hand the structure itself of the Lorentzian
theory was easily criticised by physicists having an antirealistic inclination. For
example Heisenberg wrote: “Since all systems of reference that are in uniform
translation motion with respect to each other are equivalent for the description
of nature, there is no meaning in the statement that there is a substance, the
ether, which is at rest in only one of these systems.” [15-15] The diffusion of
the Copenhagen formulation of quantum mechanics, after the mid twenties,
brought the large majority of physicists to a philosophical unification on an
idealistic basis. And in such a frame of ideas the relinquishment of ether and the acceptance of subjectivistic relativism became the great and stable fashion of the XXth century.

In spite of the general trend Einstein's works failed to convince Lorentz that the ether was superfluous. "Lorentz defended the concept of the ether until the end of his life, and his tenacity led Einstein to introduce a new concept of the ether in 1916." [15-16]. This was Einstein's relativistic ether to which our section 4 was devoted.

When Lorentz received the final formulation of the General Theory of Relativity he still believed that Einstein's new theory could be reconciled with the concept of the ether at rest. He tried to convince Einstein, by writing to him a long letter (1916), which is of the highest interest as it gave rise to a friendly but firm confrontation between two giants of the scientific thought on the foundations of modern physics.

"We can imagine performing an experiment with two perfectly conducting wires, which are stretched around the earth at the equator, each one being closed on itself. In order not to risk the “derailment” of the electromagnetic waves (caused by the earth curvature) we can use one wire with a concentrical conducting wrapper, instead of two wires. At a certain point, A, of this “cable” closed upon itself, there should be a device which makes it produce waves, and a detector with which we observe in A the returning waves as they complete the circle. The cable and the point A should be fixed to the earth. According to what we know we could firmly predict that waves which are produced at the same time in A and which cross the circumference in opposite directions will not return at the same time in A."

[15-17]

This is an important conclusion strengthened by the fact that Einstein visibly agrees on the point. To the above Lorentz adds that among the different ways, in which one can describe this result, there are two that are particularly simple.

a. One can choose a system of coordinates I OX, OY (OZ must coincide with the axis of the earth) in such a way that in this system the transmission velocity of the waves for both crossing directions is the same: \(c_1 = c_2\). One finds then that the earth rotates in this system of coordinates.

b. One introduces a system of coordinates II, which is firmly fixed to the earth. With respect to it, there are different propagation velocities \(c_1\) and \(c_2\) for the two circulation directions: \(c_1 \neq c_2\).
In this sense one will conclude: the propagations in the cable do not behave in the same way with respect to the systems of coordinates I and II. Now, if one tries to make this somehow understandable, and to represent it in space and time, it will be really impossible to imagine that nothing exists in space or in vacuum behaving differently with respect to the systems I and II. The representation is very easy to find: in the cable there is a medium (ether) in which the waves propagate, in such a way that the propagation velocity relative to the medium is always the same.

![Diagram](image)

Figure 19. This figure with two wire rings near the Earth surface was inserted in the Einstein-Lorentz debate papers.

This medium is at rest if referred to the system of axes I, and it can move if referred to the system of axes II. If one starts from this point of view one can say that the experiment has detected the motion of the earth relative to the ether. In this way, having recognized the possibility of detecting a relative rotation, one cannot a priori deny the possibility to obtain also the effects of a translation of the same type, that is to say one should not make as a postulate the starting assumption of the theory of relativity. One must rather search for the answer in the observations. One can express the relativity principle as a fundamental hypothesis, thus still allowing the possibility (however
improbable it may be considered) that clever observations will force in the future to drop the hypothesis of relativity. One can present these considerations also in another way. One can produce standing waves in the closed cable and can observe at every instant the position of the nodes. It will then result, that they run on a circle relative to the earth. One could however limit oneself, to observe the relative motion of the nodes with respect to the earth. If however one considers, that the same rotation appears in standing waves of different length and different intensity, it becomes natural (as a figurative summary of the common cause of all these phenomena) to think of an ether, in which the standing waves have their seat.

Lorentz's conclusion was as follows: "Both the theory of relativity and your gravitational theory can remain in their full extension also according to the point of view which I defended. Only they will less impose themselves on us as the unique possibility." [15-18]

Einstein's answer was dated 16 June 1916 [15-19]. Einstein did not agree with challenging his relativity principle. Neither did he agree with Lorentz's opinion that the General Theory of Relativity could be reconciled with the concept of the stationary ether. For the first time, however, he proposed a concept of the new, nonstationary ether which would not violate the relativity principle.

Let us quote in Einstein's letter the points where he disagreed with Lorentz. "I agree with you that the general theory of relativity is closer to the ether hypothesis than the special theory. This new ether theory, however, would not violate the principle of relativity, because the state of this \( g_{\mu\nu} = \text{ether} \) would not be that of a rigid body in an independent state of motion, but every state of motion would be a function of position determined through the material processes." [15-20]

If the earth did not exist or did not rotate, the interference nodes of ring I and II would remain at rest in relation to the “fixed stars” and also in relation to each other. But the earth rotates and both node systems rotate with it, even if in a very tiny percentage, and specifically the system of nodes of I, because of the smaller distance, more than that of II. The systems of nodes I and II rotate also with respect to one another with a very small velocity.

Einstein prefers the system of \( g_{\mu\nu} \) to an incomplete analogy with a material something. This – he says - because the privileged nature of the uniform motion does not appear in these modified hypotheses of ether, while it does in the abstract system. In fact, if one starts from a region of the world with
constant $g_{\mu\nu}$, a linear substitution of the $x_\nu$ does not change the constancy of $g_{\mu\nu}$, while certainly does so a nonlinear substitution of the $x_\nu$. From this follows, that the uniform relative motion does not “produce” any gravitational field, that is to say it remains unnoticed contrary to non uniform motion. [15-21]

As we can see, Einstein rejected the existence of a stationary ether violating the relativity principle. In his idea the new ether could be introduced instead: it would respect the relativity principle because it would not be a medium with its own state of motion. The state of the new ether (its metric behaviour) would be described by tensor $g_{\mu\nu}$, and that is why Einstein wrote "state $g_{\mu\nu} = \text{ether}$".

Finally, one could ask, who won in the Einstein-Lorentz debate? In my opinion nobody did and the confrontation ended with a tie. None of the two champions of scientific rationality admitted having made a mistake, both ended the debate more convinced than ever of owning the potentiality for great discoveries. The conclusion is typical of the confrontations between high level physicists, divided only by different philosophical outlooks. Other examples are Newton vs. Leibnitz on the nature of space and time and Einstein vs. Bohr on the foundations of quantum theory.

The lack of a logical-scientific victory of Einstein over Lorentz is another reason why today the realistic outlook of the Dutch physicist is still an open possibility for the development of physics.

We should not forget Lorentz’s main reason for believing in the ether. Told with the style of Agathangelidis it is so: “From Experimental Physics it is well known that light shows a wave-like behaviour and that exactly imposes the existence of a carrier of its vibrations. This carrier or medium takes the names: luminiferous ether, classical ether or ether.” [15-22]
[15-1] Quoted by [EW, p. 115]
[15-6] [PE, p. 315]
[15-11] H.A. Lorentz,
[15-14] [TE2, pp. 229-230]
[15-15] [WH, p. 114].
[15-16] [LK, p. 32]
Chapter 16

The cosmological question

According to the Lorentz transformations the product $ct$ is similar to the space variables $x, y, z$. The analogy pushed the physicists, Einstein included, to accept a four dimensional space to represent the relativistic reality. Today it is taught that the universe is a four dimensional continuum in which space and time are interconnected. In 1907 Minkowski succeeded in formulating anew Einstein’s theory of special relativity. He did so by exploiting the concept of four dimensions: three of space and one of time. Minkowski’s four dimensional space-time helped Einstein to develop his theory of general relativity, which, eventually, he came to regard as his greatest achievement.

Cosmology is not independent of fundamental physics, as the history of XXth century science shows. In this book we have proposed new transformations of the space and time variables, the Lorentz transformations not providing a satisfactory solution in some cases. We expect that the contemporary cosmological theories may require modifications as well. For a long time the agreement between the big bang theory and the experimental data has been acceptable. Recently, however, the agreement started to deteriorate. At the European Observatory ESO on the Chilean Andes a team of astrophysicists has surprisingly discovered some galaxies “old”, that is to say very evolved and with a large mass, at a time in which the Universe was still “young”, little evolved and with an age about four times smaller than 14 billion years, considered to be the actual age. It has also been possible to understand that these galaxies have a “spiral”, or “elliptical” structure completely similar to the galaxies found in the actual universe. In other words, they are a sort of copies of the actual galaxies, but already present in very remote cosmic times in which these galaxies should not have existed according to the accepted theory.

Using the Very Large Telescope at Paranal, a team of astronomers has identified four remote galaxies, several times more massive than the Milky Way galaxy, or as massive as the heaviest galaxies in the present-day universe. Those galaxies must have formed when the Universe was only about 2,000 million years old, that is some 12,000 million years ago.

M.J. Geller and J.P. Huchra, of the Harvard-Smithsonian Center for Astrophysics, mapped all galaxies within about 600 million light years of Earth. In 1989 they announced their results, revealing what they called the “Great Wall,” a huge sheet of galaxies stretching in every direction off the region mapped. The sheet is more than 200 million light years across and 700 million light years long, but only about 20 million light-years thick.
It is very impressive that the theory presented in this book goes in the same direction as the recent observations. All the theoretical formulations of the big bang model have found it unavoidable to introduce a four dimensional space-time. These models are in a condition of shaky equilibrium, precisely because they are founded on the four dimensional space of the TGR, in turn derived from the four dimensional Minkowski space of the TSR. Thus the big bang depends strongly on that mixing of space with time which is typical of the TSR. In other words, the big bang model is in great danger of structural collapse if the future progress of physics should lead to a modification of the fourth Lorentz transformation. But in this book we have shown that the empirical evidence strongly demands that the Lorentz transformations be replaced by the inertial transformations. If this is done, however, nothing remains of the symmetry between space and time from which the notion of four dimensional space-time arose. The arena of physical events and phenomena remains the old fashioned 3D space of Descartes and Newton.

16. The cosmological question

Cosmology is not independent of fundamental physics, as the history of XXth century science has abundantly shown. In this paper we have developed the proposal (better, the necessity) of a modification of the transformations of the space and time variables between inertial reference frames, the Lorentz transformations not providing a satisfactory solution in some cases. We can expect that the contemporary cosmological model may require modifications too.

In the Lorentz transformations the space variable \( x' \) is a linear function of \( x \) and \( ct \). The linearity exists also in classical Galilean physics, with a difference between the two approaches which is numerically very small for most practical experimental situations. In fact one has

\[
x' = x - \left(\frac{v}{c}\right)ct \quad ; \quad \frac{1}{R} \left[ x - \left(\frac{v}{c}\right)ct \right]
\]

\[(16.1)\]

where

\[
\frac{1}{R} = \frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{1}{2} \frac{v^2}{c^2}
\]

Therefore the difference between the two approaches is of the second order in \( v/c \), a very small quantity in many experiments.

The situation is different for the transformation of time. One has
and it is clear that the difference between the two approaches this time can be much larger than in (16.1); indeed it is of the first order in $v/c$. In this way the real novelty of relativistic physics appears to be the presence of a space variable in the transformation of time. From the mathematical point of view one can add that the structure of the Lorentz transformations in (16.1) and (16.2) was the same. If (16.1) could be interpreted as a straight line, why not interpreting (16.2) in the same way? The reflection over this point slowly convinced people, Einstein included, to take literally the Lorentz transformations: $c \, t$ and $c \, t'$ are similar to the space variables $x$, $y$, $z$, the difference remaining only in the pseudo-Euclidean metric associated with the “time” direction. But the positive part of the analogy was impressive and pushed the physicists to accept a 4-dimensional space to represent the relativistic reality. Today it is taught that the universe is a four dimensional continuum in which space and time are not separate, but are interconnected. Space is three-dimensional, and time is one-dimensional, so together they make up the four dimensional spacetime. Spacetime is a continuum because there are no missing points in space $x$, $y$, $z$ or moments in time $t$.

In 1907 Hermann Minowski succeeded in formulating Einstein's theory of special relativity with what was then modern mathematics (tensor calculus) [16-1]. He did so by exploiting the concept of four dimensions: three of space and one of time. Using this mathematical representation, he was able to describe the positions and motions of objects such as speeding electrons as they moved through space. Minowski's four-dimensional spacetime helped Einstein to develop his theory of general relativity, which he would come to regard as his greatest achievement.

For a long time the agreement between the big bang theory and the experimental data has been excellent. Only recently the agreement started to deteriorate. Let us see a couple of examples. A research group of INAF (Istituto Nazionale di Astrofisica, Italy) has discovered the existence of highly evolved and large mass "old" galaxies, very similar to the actual ones, at a time in which the Universe was still "young" with an age only one fourth of 14 billion years. The discovery, obtained from images produced by the best telescopes existing on ground or in space, is important as up to the present time people thought that in those primitive times only young galaxies could exist with small mass and in full formative activity.

According to the accepted theories, galaxies as we observe them in the nearby universe are the result of an evolutionary process, thanks to which they have slowly grown by “assembling“ smaller galaxies which aggregated to one another simply because mutually attracted by gravity. At the end of this hierarchic process from
“small” galaxies containing about one billion stars would have formed the large galaxies, containing up to hundred billion stars, which we observe in the present time. In this picture, in the past the universe was populated by galaxies young, small and with little mass, while the present day universe contains galaxies arrived at the end of the hierarchical process of growth, and then old, big and with a much larger mass.

To verify the validity of this theory the research team has studied carefully the properties of galaxies very far away in space and consequently much backward in time and in the evolutionary process of the universe. The group was led by Andrea Cimatti, INAF of Florence, and included scientists of INAF of Bologna, Trieste and Roma and of Padua University.

From images obtained with the largest telescope existing on Earth (the European Observatory ESO on the Chilean Andes) the team has surprisingly discovered some galaxies “old”, that is to say very evolved and with a large mass, at a time in which the Universe was still “young”, little evolved and with an age about four times smaller than 14 billion years, considered to be the actual age. Thanks to complementary data obtained with the Hubble Space Telescope (NASA/ESA) it has also been possible to understand that these galaxies have a “spiral”, or “elliptical” structure completely similar to the galaxies found in the actual universe. In other words, they are a sort of copies of the actual galaxies, but already present in very remote cosmic times in which these galaxies should not have existed according to the accepted theory,

Using the Very Large Telescope at Paranal, the team of Italian astronomers has identified four remote galaxies, several times more massive than the Milky Way galaxy, or as massive as the heaviest galaxies in the present-day universe. Those galaxies must have formed when the Universe was only about 2,000 million years old, that is some 12,000 million years ago [16-3].

The unexpected existence, in so young a universe, of these cosmic objects already perfectly formed, practically equal to other objects observed in the nearby “contemporary” universe poses a problem. It could demonstrate that the formation mechanisms of galaxies, the true building blocks of the Universe, in remote times have been much faster than predicted by the commonly accepted theory of hierarchic formation. In turn this rises serious questions on our effective knowledge of the evolutionary processes of the universe and of the structures (such as galaxies) which characterize it, opening new horizons for research in astrophysics and cosmology. In the present writer’s opinion, the discovery of old galaxies in the young universe represents yet another step away from the traditional big bang description of the universe.

The lack of uniformity is confirmed by other studies of the sky. In August 2007 radio astronomers announced to have found the biggest hole ever seen in the universe [16-3]. Radio sources track the distribution of mass in the universe. They are the signposts for galaxies and, of course, for clusters of galaxies as well. A group of astronomers of the University of Minnesota (Minneapolis) was puzzled by the fact that photons of the cosmic microwave background (CMB) coming from a region in
the sky in the direction of the constellation Eridanus are colder than from any other
direction. When they pointed their radio telescopes towards Eridanus they had a
surprise. They saw little or no radio sources in a volume that is about a billion light
years in diameter. The lack of radio sources means that there are no galaxies or
clusters in that volume. The void, which is about 6 billion to 10 billion light years
away, is much larger than any found before. Until now optical surveys found many
voids in the distribution of matter in the sky, but none with linear dimensions larger
than about 260 million light years. This means that the new void is about 40 times
larger in volume than the previously largest one. Presumably it is the cause of the
slightly colder temperature of the CMB coming from the Eridanus direction. The
cosmological difficulties arising from such a huge void become clear if we remember
that galaxies rarely move much faster than a thousand kilometers per second, about
one-three-hundredth as fast as the velocity of light. Therefore a galaxy born from the
big bang could have moved only about one-three-hundredth of 14 billion light years,
or about 50 million light years. But if one started with matter spread smoothly
through space, and if one can move it only 50 million light years, one cannot build up
empty spaces as vast as that found in Eridanus. For such an hole to form, matter must
have moved much more.

One should recall the important work done by H. Arp [16-4] who has
collected an impressive number of “anomalous redshifts”, due to galaxies and quasars
in the sky which can be shown not to be at their redshift distance. Very interesting
already from the title, is also the review paper by Narlikar [16-5]. On the average,
however, the redshift distances should be correct, otherwise one should imagine that
regular structures, such as the recently discovered Great Wall, emerge by pure
chance from disordered data. M.J. Geller and J.P. Huchra, of the Harvard-
Smithsonian Center for Astrophysics, mapped all galaxies within about 600 million
light years of Earth. In 1989 they announced their results, revealing what they called
the “Great Wall,” a huge sheet of galaxies stretching in every direction off the region
mapped. The sheet is more than 200 million light years across and 700 million light
years long, but only about 20 million light-years thick.

At this point one would need a credible mechanism for producing the
observed redshifts. La Violette [16-6] has studied the consequences of tired light
mechanisms and concluded that various empirical tests speak for tired light and
against the big bang explanation. There are, however, two standard objections against
the tired light idea. First, the frequent scattering of the photon smears out the
coherence of the radiation, and so all images of distant objects should look blurred,
and they are not. Second, the scattering effect and the consequent loss of energy is
frequency dependent, thus making (for hydrogen) a 21 cm redshift different from the
redshift of the line Lyman $\alpha$, say. Experimentally instead the redshift is the same
from visible radiation to radio waves.
We can add that some recent evidence suggests that the universe is not expanding at all, but the galaxies are each moving in space in a radial direction away from the origin in or near the Virgo supercluster. Naturally the big bang theory has no center of expansion. A much used method for providing an intuitive understanding of big bang is the analogy with the surface of an inflating rubber balloon covered with dots, adding that the real world is however the three dimensional surface of a four dimensional sphere. It is easy to understand that a balloon surface cannot have any center in the surface itself. Therefore, the big bang cannot have any center of explosion, because this would violate the big bang theory itself. But the Virgo expansion center seems to prove just the opposite! Thus we see that quite an amount of evidence points to a universe very different from the usual picture provided by the big bang model. It is for us very impressive that the theory presented in this book goes exactly in the same direction as the recent observations. Let us see why.
It is very impressive that the theory of space and time presented in this book goes exactly in the same direction as the recent astrophysical observations. Let us see why. The adoption of the four dimensions was essential for the acceptance of the model. In the ordinary three-dimensional space a great cosmic explosion would produce matter and throw it in all directions. Later, galaxies would appear from the aggregation of the primordial matter and would have different radial velocities. This process would generate an universe with an easily recognizable structure. At the centre there would be a huge empty region, the “crater” of the explosion; an intermediate region containing the galaxies; an external region containing only radiation. From any conceivable position in the intermediate region, we would see a universe very different from the basically homogeneous one predicted by the big bang theory. This point was made in a very sharp and convincing way by G. Bonali [16-7].

All the theoretical formulations of the big bang model have found it unavoidable to introduce a four dimensional spacetime. We should then stress that from a conceptual point of view these models are not at all in a condition of strength, but of shaky equilibrium, precisely because they are founded on the four dimensional space of the TGR, in turn derived from the four dimensional Minkowski space of the TSR. Thus the big bang depends strongly on that mixing of space with time which is typical of the TSR. In other words, the big bang model is in great danger of structural collapse if the future progress of physics should lead to a modification of the fourth Lorentz transformation. But this is exactly what I have shown to be necessary.

In this book I have pointed out that the empirical evidence strongly demands that the Lorentz transformations be replaced by the inertial transformations. If this is done, however, nothing remains of the symmetry between space and time from which the notion of fourdimensional spacetime arose. Therefore it would make no sense to insist on multidimensional curved notions of space and time. As arena of physical events and phenomena will probably remain the old fashioned 3D space of Descartes and Newton.

[16-1] H. Minkowski, Space and Time, in: [PR, pp. 73-91]
[16-7] See the book [GB].
Chapter 17

Superluminal signals: \( e_1 = 0 \)

Some physicists say that the third commandment of relativity is “do not go faster than light”. There were several good reasons for adopting such a restriction, but by far the best one was the necessity to avoid a causal super-paradox against which it would have been useless even the well established trend of throwing away increasingly large pieces of “good sense.” Simply stated, the causal paradox of the TSR can be presented as follows. Two travelers, \( A \) and \( B \), are moving in space aboard their respective spaceships: we can imagine \( B \) faster than \( A \) and both moving with constant velocity. Something happens in the B ship, the main computer goes afire and some important data is irreversibly lost. By those future times the technology of superluminal signals (SLS) is available in practically all space ships. Therefore \( B \) sends to \( A \) the request of urgent reply to a complicated technical question and uses a SLS to obtain the answer as soon as possible. The first spacetraveler finds quickly the information needed and communicates it by sending a new SLS to \( B \). Now comes the problem. The kinematics of the TSR applied to superluminal propagations of the type we are considering implies in some cases an exchange between past and future. In other words, it can happen that the \( B \)-time of arrival of \( A \)’s answer to \( B \) precedes the \( B \)-time of departure of the question. But if this happens \( B \) has already the answer to his badly needed question and could decide not to send the question to \( A \). This would be a terrible situation for the TSR because it is implied that \( A \) has answered a question he never received. He has done something (sent the answer to \( B \)) without being prompted to do so by any piece of information.

Different developments make the existence of superluminal propagations possible. For example, it has been shown that Maxwell’s equations have solutions representing electromagnetic fields propagating with arbitrarily large group velocity. But of course also this matter will finally be decided by experiments.

Concerning synchronization, the TSR made the simplest choice by postulating the invariance of the oneway velocity of light and applying the same synchronization to all inertial frames. However, simplicity in one respect can give rise to complication in another, as it happens with the SLS, which are excluded in the TSR. We propose an alternative approach, based on the ET, according to which the one way velocity of light is physically isotropic only in \( S_0 \). Therefore the Einstein synchronization has to be implemented in \( S_0 \) in all cases and, once
this is done, clocks at rest in $S_0$ measure the true physical time. Therefore $S_0$ is the privileged system in which the Lorentz luminiferous ether is at rest. It is natural to assume that electromagnetic perturbations (including SLS) are vibrations of this medium. From such a point of view a SLS could not possibly pass near clocks at rest in $S_0$ showing a decreasing time as, even for superluminal velocity, a signal requires a finite positive time to cover a finite distance. The roots of the causal paradoxes are thus seen to lie in a much too symmetrical treatment of all inertial systems. Assuming that SLS propagate in a Lorentz medium at rest in $S_0$ excludes negative velocities relative to $S_0$ and thus limits the velocities relative to other inertial systems to such values that do not become negative when transformed to $S_0$. In this way, that is by abandoning relativism and admitting the existence of a privileged reference frame, causal paradoxes disappear. Among all the possible “equivalent” theories there is one for which not even the kinematical cutoff is needed, the theory of inertial transformations.

17. Superluminal propagations: $e_1 = 0$

Recently some experimental evidence for the existence of signals propagating with a velocity larger than $c$ (“superluminal”) started to accumulate. In this section we will see that the causal paradoxes generated in the theory of special relativity (TSR) by the eventual existence of the superluminal signals (SLS) can be solved by considering the set of the “equivalent” theories (set to which the TSR belongs), differing from one another in the parameter $e_1$ only. In fact one can show that the elimination of the causal paradoxes is possible in one way only, by assuming absolute simultaneity in the transformation of time ($e_1 = 0$). As we saw in previous sections, this conclusion is not only compatible with the classical experiments on which the STR is based, but is even required by other independent considerations.

Two developments make the existence of superluminal propagations possible, perhaps even probable. From the theoretical point of view it has been shown that superluminal solutions of the relativistic equations exist [17-1]. For example, Maxwell’s equations have solutions representing electromagnetic waves propagating with arbitrarily large group velocity, a rather unexpected result.

From the experimental point of view, evidence of structures propagating with velocity larger than $c$ has been found in different areas, such as:

1. Tunnelling photons. Ever since 1992 it had been shown in Cologne [17-2] that tunneling wave packets can move with superluminal group velocities inside
the barrier. The result was confirmed by independent experiments carried out in Berkeley [17-3]. The phenomenon had been predicted theoretically.

2. Microwave propagations. Microwave pulses have been observed to propagate in open air with a velocity of about $2c$ [17-4]. The effect was present when the detector was displaced laterally with respect to a launcher 90 cm away and was attributed to evanescent waves which decay over distances of a few wavelengths.

3. X-shaped waves. Bessel pulses with an X-shaped structure have been predicted theoretically and observed experimentally [17-5], [17-6]. Their superluminal velocity is $c / \cos \theta$, where $\theta$ is the cone angle of the Bessel beam.

4. Astrophysics. If quasars are taken to be at redshift distances (big bang model), then superluminal expansions up to 45 $c$ have been observed [17-7]. But of course Arp showed that the redshift quasar distances are unreliable [17-8]. However, even within our Milky Way (where distances are well established) there is evidence of something moving with superluminal velocity [17-9]. Furthermore there are the M87 ejections (blue knots propagating at velocity $5-6 c$!) whose distance does not depend on red shift, but was obtained from Cepheids, planetary nebulae, apparent size of galaxy, etc. This distance is somewhere around 50 million light years. [H. Arp, private communication]

After stressing that superluminal signal velocities do exist, the causal paradox of relativity was discussed by Nimtz and Haibel [17-10] who could show that it was prevented from becoming effective by the finite duration of the signal in the experiments performed by the Cologne group.

More difficult is the case of the paper by Garrison et al. [17-11] where the question whether the existence of superluminal signals generates a conflict with special relativity is answered by saying that no disagreement has been found with Maxwell’s equations. These equations being consistent with special relativity, these authors argue, no inconsistency can exist with the theory. But the simple minded argument is debatable.

The general solution advocated by Recami [17-12, 17-13] is based on his “reinterpretation rule”: a superluminal particle which appears to us to propagate forward in time is actually an antiparticle going in the opposite time direction, from the future towards the past. Apply this idea systematically, Recami says, and the causal paradoxes will disappear. That this solution is too simplistic to be acceptable can be shown with some counterexamples. If they really exist, in some time of the future it will be possible to do with tachyons what is normally done with ordinary particles: prepare beams, control the times of emission with shutters, and so on. We give below three arguments against the reversal of the time arrow.

1. Consider a beam of tachyons interacting with a target of some ordinary material: those undergoing scattering will be deviated in many different
directions, since it is impossible to control the individual behaviour in a collision. The situation would be as shown in Fig. 21. The “reinterpretation rule” would have to say that the (anti)tachyons are really coming from all directions on the right (from the future!) but that they move parallel to one another as consequence of the collisions with the atoms composing the slab. These tachyons should be too well behaved, as their collisions would clearly contradict the basic rules of probability!

Figure 21. A parallel beam of tachyons is dispersed in many directions after scattering on the atoms composing a slab.

2. The famous causal paradox of relativity can be complicated at will. Consider two physicists, Fred (F) and Gus (G), and assume that F is moving away from G through space at high speed, although lower than the speed of light. Assume that both are able to produce superluminal signals at chosen times. At a certain moment F, who noticed an accidental deletion from his encyclopedia, uses a set of suitably organised SLS’s to ask G the following question: “What is the ninth word in the eighty-second line on page 1176 of volume XIV of the British Encyclopaedia?” G diligently looks for the word in question and answers F with another organised SLS: “The word you want is ‘usucaption’.” We know that relativity theory predicts that the answer from G will reach F before he has sent his question. His curiosity satisfied, F may, however, decide not to send his question to G. But if G does not receive the query from F, how does he answer such a complicated question? Obviously we are facing a logical impossibility.
Now Recami should try to solve the paradoxical situation by saying that actually it was the detector of F that produced particles going backward in time to the source of G. But a simple change of the direction of time does not cancel the information stored: how is this possible if not even F (let alone his detector) knew the transmitted word?

3. One of the arguments used in favor of propagations towards the past is that antiparticles are to be considered ordinary particles traveling backwards in time, as suggested by Feynman [17-14]. Hence a positron, for instance, would be only a normal electron going from the future towards the past. This opinion is, however, hardly consistent with the circumstances under which the positron was discovered, as recounted by Alvarez [17-15]. Most people would say that the discovery of the positron rested on the observation that an electron-like track in a magnetic cloud chamber bent “the wrong way”. But that would not be correct, since others had previously seen electron-like tracks curving that way in cloud chambers and the effect was always attributed to electrons going in the opposite direction in space. Anderson's discovery of the positron rested entirely on the fact that he knew which direction his positron was going; he placed a lead plate in his cloud chamber and saw the particle lose energy and “curl up” after going through the plate. (see Fig. 22)

If the particle were traveling from the future (above the slab) to the past (below) it would have to gain energy in crossing the lead plate. Given the complexity of the multiple interactions with atoms in the metal, the probability of an energy gain is extremely low, and its regular observation would be entirely inconsistent with the known laws of probability and thermodynamics. Hence a positron is not a normal electron going to the past.
Many observers had seen particles that were consistent with the positron hypothesis, but Anderson was the first one to reject all other alternatives. Given the inertial frames $S_0$ and $S$ one can set up Cartesian coordinates and make four assumptions which have been seen in the seventh section. The frame $S_0$ turns out to have a privileged status in all theories satisfying the first two assumptions, with only one exception, the TSR. Two further assumptions based on direct experimental evidence can be added:

(v) The two way velocity of light is the same in all directions and relative to all inertial systems;

(vi) Clock retardation takes place with the usual velocity dependent factor $S$ [see (6.3)] when clocks move with respect to the isotropic reference frame $S_0$.

Together these six conditions were shown to imply the (“equivalent”) transformations of the space and time variables from $S_0$ to $S$ given by (7.2). As a consequence of the equivalent transformations the one way velocity of light $c_1(\theta)$ relative to the moving system $S$ for light propagating at an angle $\theta$ from the velocity $\vec{v}$ of $S$ relative to $S_0$ (“absolute” velocity of $S$) turns out to be given by (7.3) and (7.4). For the two way velocity of light one has, of course, in all directions, $c_2(\theta) = c$.

The transformations inverse of (7.2) are

\[
\begin{align*}
x_0 &= (R - e_1 v) x + \frac{\vec{v} t}{R} \\
y_0 &= y \\
z_0 &= z \\
t_0 &= (t - R e_1 x) / R
\end{align*}
\]

The equivalent transformations between two inertial systems $S$ and $S'$ moving relatively to $S_0$ with velocities $\vec{v}$ and $\vec{v}'$, respectively, can be obtained by substituting (17.1) into the Eq.s (7.2). The result is published [17-16]. The transformations (7.2), (17.1) contain only a free parameter, $e_1$, the coefficient of $x$ in the transformation of time. We show below that $e_1$ can be fixed by choosing a clock synchronisation method. The Theory of Special Relativity (TSR) is
recovered for \( e_1 = -v / c^2 R \) (giving \( \Gamma = 0 \) and, of course, \( c_1(\theta) = c \)). Different choices of \( e_1 \) imply different theories of space and time which have been shown to be empirically equivalent to a large extent. Michelson type experiments, Doppler effect, aberration, occultations of Jupiter satellites, and radar ranging of planets were shown to be insensitive to the choice of \( e_1 \). It was therefore concluded that Römer’s and Bradley’s observations were not truly one way measurements of the velocity of light as \( c_1(\theta) \) depends on \( e_1 \) but they never needed to specify the value of \( e_1 \).

We will now use the equivalent transformations to discuss that exchange of superluminal signals which gives rise to the typical causal paradox in the STR. Only the case \( e_1 \leq 0 \) in (17-1) will be presented, but we checked that the case \( e_1 > 0 \) can be treated in a very similar way and leads to exactly the same conclusions. We will show that the essence of the causal paradox lies in the impossible requirement that a superluminal propagation may overtake a set of clocks marking a progressively decreasing physical (then, not conventional) time. The particular choice \( e_1 = 0 \), which, as we saw, is selected by several phenomena in which the acceleration has a role, remains highly preferable also in the present context, being the only one not leading to the causal paradox.

With reference to Fig. 23 suppose a localized superluminal signal \( \sigma_1 \) is emitted by a device \( \Sigma_0 \) at rest in the origin of \( S_0 \) at time \( t_0 = t = 0 \) and propagates along \(+x_0\) according to the equation \( x_0 = u_0 t_0 \) with \( u_0 > c \). Its position \( x \), seen from \( S \), is given by the first Eq. (7.2), which becomes

\[
x = \left[u_0 - v \right] t_0 / R.
\]

Inverting the previous equation one has \( t_0 = Rx / (u_0 - v) \). Therefore a device \( \Sigma \) (detector and source) placed in the point at rest on the \( x \) axis of \( S \) with coordinate \( x=x_1>0 \) is reached at a time \( t_{01} \) given by

\[
t_{01} = \frac{R}{u_0 - v} x_1
\]

As \( u_0 > c > v \), \( t_{01} \) cannot be negative and \( t_{01} = 0 \) only if \( u_0 \) is infinite. Given \( x_0 = u_0 t_0 \) and Eq. (17.3), at time \( t_{01} \) the signal has a position in \( S_0 \) given by

\[
x_{01} = \frac{u_0 R}{u_0 - v} x_1
\]
Figure 23. a) The superluminal signal $\sigma_1$ emitted by $\Sigma_0$ in the origin of the system $S_0$ propagates along $+x_0$ until absorbed by a device $\Sigma$ at rest in the point $x_1 > 0$ of the system $S$; b) immediately after absorbing $\sigma_1$, $\Sigma$ emits a new superluminal signal $\sigma_2$ which propagates along the $-x$ axis until it is absorbed by $\Sigma_0$. The process is represented from the point of view of the observers in $S_0$.

When the point $x_1$ is reached the clock at rest in $S$ near $x_1$ marks the time $t_1 = R t_{01} + e_1 (x_{01} - v t_{01})$, which owing to (17.3) and (17.4) becomes

$$t_1 = R \left[ \frac{R}{u_0 - v} + e_1 \right] x_1$$

(17.5)

Of course this is the same as saying that $t_1 = x_1 / u$. Notice that at a critical superluminal velocity $\tilde{u}_0$ given by
\[ \tilde{u}_0 = v - \frac{R}{e_1} \]  

(17.6)

one has \( u = \infty \) and, of course, \( t_1 = 0 \). As a particular case, using the relativistic value of \( e_1 \), Eq. (17.6) gives the superluminal velocity \( \tilde{u}_0 = c^2 / v \). Since \( x_1 \) could be any point, if \( \sigma_1 \) propagates with constant velocity \( \tilde{u}_0 \) it will overtake clocks at rest in \( S \) all showing time \( t = 0 \). Furthermore, if \( u_0 > \tilde{u}_0 \), \( \sigma_1 \) will progressively overtake clocks showing a decreasing time. This needs not having to do with causal paradoxes, being merely an artifact of the \( x \) dependence of the (conventional) synchronization of clocks in \( S \). Therefore there is no reason to restrict \( u_0 \) to values not exceeding \( \tilde{u}_0 \) and all superluminal velocities are acceptable relatively to the frame \( S_0 \).

The signal \( \sigma_1 \) is detected by \( \Sigma \) in point \( x_1 \) at the time \( t_1 \) of \( S \). At the same time a second signal \( \sigma_2 \) leaves \( \Sigma \) moving along \(-x\) with the superluminal speed \( w \) relative to \( S \). Its equation of motion is

\[ x = x_1 - w(t - t_1) \]  

(17.7)

with \( t \geq t_1 \) and \( w > c \). The problem is to find the time at which \( \sigma_2 \) reaches the device \( \Sigma_0 \) in the origin of \( S_0 \) which emitted the first signal. It follows from the inverse transformations (17.1) that the origin of \( S_0 (x_0 = 0) \) satisfies in \( S \) the equation of motion

\[ x = -\frac{v}{R(R - e_1v)} t \]  

(17.8)

Relatively to \( S \) \( \sigma_2 \) has to be faster than the origin of \( S_0 \), if it is to reach it. Therefore

\[ w > \frac{v}{R(R - e_1v)} \]  

(17.9)

The instantaneous overlapping of the propagations described by (17.7) and (17.8) will take place at a time \( t_2 \) of \( S \) at which the positions coincide:

\[ -\frac{v}{R(R - e_1v)} t_2 = x_1 - w(t_2 - t_1) \]

whence
The event \((x_2, t_2)\) takes place in the origin of \(S_0\) and therefore satisfies in \(S\) Eq. (17.8) with \(x = x_2\) and \(t = t_2\). Then Eq. (17.10) gives

\[
x_2 = \frac{-v}{wR(R - ve_1) - v} \left( x_1 + wt_1 \right)
\]  

(17.11)

Considering the event \((x_2, t_2)\), we insert (17.10) and (17.11) in the inverse transformation of time (17.1) to get

\[
t_{02} = \frac{R}{wR(R - ve_1) - v} \left( x_1 + wt_1 \right)
\]  

(17.12)

It is interesting to notice that the \(w\) dependence of \(t_{02}\) is all explicit in (17.12) as none of the quantities \(R, e_1, x_1, t_1\) depends on \(w\). Therefore it is not difficult to show that

\[
\frac{\partial t_{02}}{\partial w} = \frac{-vR t_1 - R^2(R - ve_1)x_1}{\left[wR(R - ve_1) - v\right]^2} < 0
\]  

(17.13)

the last inequality being due to \(x_1, t_1 > 0\) and \(e_1 < 0\). Therefore \(t_{02}\) is a decreasing function of \(w\), as one expects from the physical point of view.

Now, Eq. (17.3) can be inverted to get \(x_1\) in terms of \(t_{01}\) and this allows one to use Eq. (17.5) to get also \(t_1\) in terms of \(t_{01}\). When this is done one gets

\[
x_1 + wt_1 = \frac{wR^2 + (u_0 - v)(1 + Rwe_1)}{R} t_{01}
\]  

(17.14)

Therefore Eq. (17.12) becomes

\[
t_{02} = \frac{wR^2 + (u_0 - v)(1 + Rwe_1)}{wR^2 - v(1 + Rwe_1)} t_{01}
\]  

(17.15)
Also in the present case one could consider a critical velocity of $\sigma_2$, $\tilde{w}$, for which the $S_0$ time shown by the clocks overtaken by $\sigma_2$ does not increase, so that $t_{02} = t_{01}$. In such a case the fraction in the right hand side of (17.15) has to equal unity. Looking at the structure of numerator and denominator it is obvious that this happens if $1 + R \tilde{w} e_1 = 0$, whence

$$\tilde{w} = - \frac{1}{R e_1} \quad (17.16)$$

Notice that $\tilde{w} > 0$, due to $e_1 < 0$. Furthermore, using the relativistic value of $e_1$, eq. (17.16) gives again the superluminal velocity $\tilde{w} = c^2 / v$ as a particular case. There is however a profound difference between $\tilde{u}_0$ and $\tilde{w}$, as the superluminal signal $\sigma_2$ moving with velocity $\tilde{w}$ would overtake clocks at rest in $S_0$ showing a constant physical time. Furthermore, for $w > \tilde{w}$, $\sigma_2$ would similarly “see” time running backwards. But, clearly, the time $t_0$, which is not conventional but real, cannot run backwards in any conceivable situation and we must impose to the superluminal velocity the “arrow of time” condition

$$w > - \frac{1}{R e_1} \quad (17.17)$$

which, being equivalent to $1 + R \tilde{w} e_1 = 0$ produces a numerator and a denominator in (17.15) such that the fraction is larger than unity, ensuring that time will always be seen to run in the right direction in $S_0$. If (17.17) were not satisfied, $t_{02}$ would belong to the past of $t_{01}$ ($t_{02} < t_{01}$) which is of course physically impossible.

The conclusion is that if the arrow of time condition (17.17) is satisfied the second superluminal signal comes back to the origin of $S_0$ at $t_{02} > 0$, that is in the future of the time $t_0 = 0$ at which the first signal left it. The causal paradox is thus solved.

Notice that $x_1$ is fixed by the experimental setup and has nothing to do with the superluminal signals. Therefore, if in (17.15) we substitute (17.3) and set no limit on $w$, allowing the superluminal velocities to go to infinity and their product $u_0 w$ to grow faster than anything else, we get

$$t_{02} \approx - \frac{e_1 R x_1}{R - v e_1} \quad (17.18)$$
which is negative (i.e., paradoxical) due to $e_1 < 0$. Therefore the arrow of time condition (17.17) is truly fundamental for overcoming the causal paradox.

To the previous considerations one could object, however, that it is not pleasant that the ad hoc condition (17.17) is assumed a posteriori in order to get rid of a paradoxical conclusion. The objection is acceptable, and once more the theory of the inertial transformations shows its superiority by avoiding the causal paradox without extra assumptions. In fact, if $e_1 = 0$ Eq. (17.15) reduces to

$$t_{02} = \frac{wR^2 - v + u_0}{wR^2 - v} t_{01}$$  \hspace{1cm} (17.19)

which implies $t_{02} > t_{01}$, as the fraction in (17.19) is larger than unity, given that $u_0 > c$. In this case no extra condition is needed, as Eq. (17.17) for $e_1 \rightarrow 0$− becomes $w < +\infty$. Furthermore the SLS would not see time running backwards in any inertial system, as $e_1 \rightarrow 0$− in (17.6) leads to $\tilde{u}_0 = +\infty$, which is the same as saying that for all finite velocities time $t$ is seen to run in the right direction.

In the equivalent transformations (7.2) there is an important difference between the inertial systems $S_0$ and $S$. In $S_0$ clock synchronisation is dictated by a physical condition, the (assumed) isotropy of the propagation of light. In $S$, instead, clock synchronization is considered a convention, based on conveniency. The TSR made the simplest choice by postulating the invariance of the oneway velocity of light and applying the same synchronization to all inertial frames. However, simplicity in one respect can give rise to complication in another, as it happens with the SLS, which are excluded in the TSR, owing to the causal paradoxes. In the present paper we proposed an alternative approach based on the equivalent transformations (7.2). The oneway velocity of light is physically isotropic only in $S_0$. Therefore the Einstein synchronization has to be implemented in $S_0$ in all cases and, once this is done, clocks at rest in $S_0$ measure the true physical time. This indicates that $S_0$ is the privileged system in which the Lorentz luminiferous ether is at rest. It is natural to assume that electromagnetic perturbations (including SLS) are vibrations of this medium. From such a point of view a SLS could not possibly pass near clocks at rest in $S_0$ showing a decreasing time as, even for superluminal velocity, a signal requires a finite positive time to cover a finite distance. The roots of the causal paradoxes are thus seen to lie in a much too symmetrical treatment of all inertial systems. In other words, in the XXth century people have believed too much in the principle of relativity. Assuming that SLS propagate in a Lorentz medium at rest in $S_0$ excludes negative velocities relative to $S_0$ and thus limits the velocities relative to other inertial systems to such values that do not become negative when transformed to $S_0$. In this way causal paradoxes disappear.
But there is more. Given two generators of SLS we consider them equal if, when placed in the same inertial system side by side, they can generate at the same time two SLS (e.g., moving on parallel lines, which arrive simultaneously in two detectors placed at the same distance in front of them. After checking in this way their equality, we can now place the two generators at rest in the inertial system $S$, facing one another at some distance, and use them to carry out the experiment described in [17-17]. The condition that the pulses emitted by the two generators have the same velocity in $S_0$ is clearly satisfied, even if one moves parallel and the other one antiparallel to the absolute velocity of its generator, as they propagate in the Lorentz ether independently of source velocities. As it has been shown, it then becomes possible to measure the absolute velocity of the laboratory in which such an experiment is carried out.
[17-8] Si veda il libro HA.
Chapter 18

Weak relativity

The inertial transformations of the space and time variables, based on absolute simultaneity, imply that a privileged (isotropic) inertial reference system exists. One can show, however, that it is possible to resynchronize clocks in all inertial frames in such a way as to select a different, arbitrarily chosen frame as "privileged". Such a resynchronization of clocks (ROC) does not modify any empirical consequence of the theory, which is thus compatible with a new form of relativity principle. Einstein based the theory of special relativity on two principles which together lead necessarily to the Lorentz transformations. In an important sense we can consider Einstein’s relativity as a strong principle. When it says that the physical laws “are not affected” by a change of reference system, it requires the laws of nature to have exactly the same form in all inertial reference frames.

From such a point of view the inertial transformations, based on absolute simultaneity and alternative to the Lorentz transformations, have milder implications. For example, Buonaura showed that Maxwell’s equations, transformed from the isotropic inertial system \( S_0 \) (where they retain the usual form) to another inertial system \( S \), acquire a generalized form, which depends on the velocity of \( S \) relative to \( S_0 \) (“absolute velocity”). From such a point of view Maxwell’s equations are “affected” by a change of reference frame given that the velocity of the frame to which they are referred is modified. In previous chapters we saw six independent proofs that the theory of the physics of space and time has to be based on \( e_1 = 0 \). The main physical question would be how to detect the privileged system. After many failed attempts I could finally prove that it is not possible to give a positive answer to such a question, in agreement with other authors. There is a well defined way to resynchronize the clocks of the universe (that is, the clocks of all inertial frames) which leads from the given privileged system to another, arbitrarily chosen, "privileged" inertial system, where the privilege is the isotropy of space, e.g. with respect to the propagation of light. Relativism pushed out of the door comes back through the window. A possible name to describe the theory of the inertial
transformations becomes thus "weak relativity". In fact, one can distinguish two formulations of the relativity principle: (1) **Strong relativity**, according to which the laws of physics are exactly the same in all inertial systems: this is Einstein’s formulation; (2) **Weak relativity**, stating merely the impossibility to measure the absolute velocity of the Earth. This principle does not demand necessarily the validity of the Lorentz transformations and opens a logical space for new theories. This formulation is essentially the original one given by Galilei extended, of course, to all physical phenomena.

Experimental evidence shows (qualitatively) that absolute velocities exist in nature. The weak relativity principle accepts this, but maintains that they remain not measurable. In spite of this conclusion I must insist that the statements made in the rest of the book are correct. The theory with the free $e_1$ applied to linear accelerations and to the Sagnac effect shows that only $e_1 = 0$ gives a rationally acceptable formulation. Similarly, only with $e_1 = 0$ one can obtain a reasonable description of aberration. The paradoxes of the special theory of relativity disappear if $e_1 = 0$. The growing evidence for superluminal signals can easily be accommodated if $e_1 = 0$, while it is incompatible with standard relativity due to the presence of a famous causal paradox. Therefore the best theory of space and time seems to be the one based on absolute simultaneity.

Nevertheless our results may seem somewhat contradictory. On the one hand they point to a theory of space and time in which such conceptions as absolute velocity, privileged frame and absolute simultaneity have a central role, while, on the other hand, a form of relativity comes back in the arbitrariness of the choice of the "privileged" frame. We must stress that both sides of the contradiction ($e_1 = 0$ and weak relativity) are absolutely correct if the assumptions made are correct. Velocity (and nothing else) is seen to be responsible for the differential retardation effect. Of course it must be an absolute velocity! Hidden behind the relativism of Einstein's theory there was a physically active background that now comes out in the clear, even though it remains (for the moment) impossible to describe it numerically.
18. **Weak relativity**

The inertial transformations of the space and time variables, based on absolute simultaneity, imply that a privileged (isotropic) inertial reference system exists. In this final chapter we show, however, that it is possible to resynchronize clocks in all inertial frames in such a way as to select a different, arbitrarily chosen frame as "privileged". Such a resynchronization of clocks (ROC) does not modify any empirical consequence of the theory, which is thus compatible with a new form of relativity principle, weaker than adopted by Einstein in the Theory of Special Relativity (TSR). From the point of view of the inertial transformations the validity of weak relativity appears accidental, more than fundamental. It would be enough to discover a very small noninvariance of the two way speed of light to make the whole game of resynchronization impossible.

Einstein based the theory of special relativity on two assumptions that were formulated as follows [18-1]:

1. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion.
2. Any ray of light moves in the “stationary” system of coordinates with the determined velocity \( c \), whether the ray be emitted by a stationary or by a moving body.”

The presence of the “stationary” system in the second assumption is only formal as the application of the first assumption (the relativity principle) to the velocity of light leads immediately to the extension of the second assumption to all inertial frames. As it is well known, the two principles together lead necessarily to the Lorentz transformations. In an important sense we can consider Einstein’s relativity as a “TSRong” principle. When it says that the physical laws “are not affected” by a change of reference system, it requires the laws of nature to have exactly the same form in all inertial reference frames.

From such a point of view the inertial transformations, based on absolute simultaneity and alternative to the Lorentz transformations [18-2], have milder implications. For example, Buonaura [18-3] showed that Maxwell’s equations, transformed from the isotropic inertial system \( S_0 \) (where they retain the usual
form) to another inertial system $S$, acquire a generalized form, which depends on the velocity of $S$ relative to $S_0$ (“absolute velocity”). From such a point of view Maxwell’s equations are “affected” by a change of reference frame given that the velocity of the frame to which these equations are referred is modified. It will be shown that in spite of this, the inertial transformations are compatible with a form of relativity principle (“weak relativity”), such that the isotropic system can be chosen arbitrarily, meaning that nothing in the theory allows one to conceive an experiment leading to the detection of the really isotropic system.

In the sections 9-13 we saw five independent proofs that the best theory of the physics of space and time has to be based on $e_1 = 0$ and cannot be the TSR. Section 17 provided the sixth independent proof of the same result.

The "inertial transformations" are obtained from Eq. (7.2), by setting $e_1 = 0$:

$$x = \frac{1}{R}(x_0 - vt_0), \quad t = R t_0$$

where $R = \sqrt{1 - \frac{v^2}{c^2}}$. In the present section we do not write down the transformations of the other two space variables when they are trivial equalities of the type $y = y_0$ and $z = z_0$, as they are in the case of Eq.s (18.1). These Eq.s imply that relative to the moving system $S$ the one way velocity of light propagating at an angle $\theta$ from the velocity $\tilde{v}$ of $S$ relative to $S_0$ (“absolute velocity”) is:

$$c_1(\theta) = \frac{c}{1 + (v/c)\cos \theta}$$

while, of course, the two way velocity of light is $c$ in all directions. The theory of the inertial transformations implies the existence of a privileged inertial system, $S_0$, in which the velocity of light is isotropic, as it is clear from (18.2) if $v = 0$. The transformations inverse of (18.1) are

$$x_0 = R \left[ x + \frac{vt}{R^2} \right], \quad t_0 = \frac{t}{R}$$
The inertial transformations between two reference frames $S$ and $S'$ moving with absolute velocities $v$ and $v'$, respectively, have a form still different from (18.1) and (18.3) and can be written

$$x' = \frac{R}{R'} \left[ x - \frac{v' - v}{R^2} t \right], \quad t' = \frac{R'}{R} t \quad (18.4)$$

where $R' = \sqrt{1 - v'^2 / c^2}$.

The inertial transformations imply absolute simultaneity: two events taking place in different points of $S$ but at the same $t$ are judged to be simultaneous also in $S'$ (and vice versa), this property being consequence of the absence of space variables in the transformation of time. Furthermore, a clock at rest in $S$ is seen from $S'$ to run slower, but a clock at rest in $S'$ is seen from $S$ to run faster. Both observers agree that motion relative to $S'$ slows down the pace of clocks. The difference with respect to the TSR does not constitute a way of choosing between the TSR and other theories, a meaningful comparison of rates implies that a clock at rest in $S'$ must be compared with at least two clocks at rest in different points of $S$, and the result depends on the synchronization adopted for the latter clocks. When a clock at rest in $S$ is compared with two or more clocks in $S'$, instead, the true, objective behaviour of the former clock is observed (naturally, if $S'$ has been chosen correctly).

Absolute length contraction is also obtained from the inertial transformations. A rod at rest on the $x$ axis of $S$ is seen in $S'$ to have end points at a common time $t_0$ such that it appears contracted by the usual factor $R$. The reasoning can be inverted by considering the rod at rest in $S'$ and observed from $S$, and using the transformations (18.1). One gets then the conclusion that the rod appears lengthened by a factor $R^{-1}$. Once more, the difference with respect to the TSR does not offer a choice between the TSR and other theories, as a meaningful comparison of rates implies that the length of a rod at rest in $S'$ must be obtained using at least two clocks at rest in different points of $S$, and the result depends on the synchronization adopted for the latter clocks. When a rod at rest in $S$ is measured using two clocks in $S'$, instead, the true, objective behaviour of the former clock is observed (again, if...
$S_0$ has been chosen correctly. We can conclude that both observers agree in saying that motion relative to $S_0$ leads to contraction. Again the discrepancy with the TSR is due to the different synchronization conventions in $S$: the length of a moving rod can only be obtained by marking the simultaneous positions of its end points, and therefore it depends on the definition of simultaneity.

The main physical question of the theory of the inertial transformations would be how to detect the privileged system. After many failed attempts I can now prove the impossibility to give a positive answer to such a question, except by using superluminal signals, if they exist (see chapter 17). There is a well defined way to resynchronize the clocks of the universe (that is, the clocks of all inertial frames) which leads from the given privileged system to another, arbitrarily chosen, "privileged" inertial system, where the privilege is the isotropy of space, e.g. with respect to the propagation of light. Relativism pushed out of the door comes back through the window. A possible name to describe the theory of the inertial transformations becomes thus "weak relativity". The skeptical reader at this point will think: "Well, after all nothing has changed and we are back to the old relativism." It is not so, first of all because the new relativistic requirements are weaker than they were before this research got started. Even more important, the Lorentz transformations must be dropped and substituted by the inertial transformations. The little word "must" holds for every person caring about rational thinking and the progress of mankind.

From the point of view of the inertial transformations the $S_0$ system is initially considered to be privileged, and the velocity of light relative to it to be isotropic. Other inertial systems ($S, S', \ldots$) are initially described as "moving" and relative to them the observers detect an anisotropic velocity of light given by Eq.s like (18.2). In the present section we will describe a process of resynchronization of clocks (ROC) and show that it is uniquely determined by the new inertial frame $S'$ chosen to replace $S_0$ as "privileged" and by the condition that absolute simultaneity should be preserved.

We assume that ROC changes the velocity of light relative to $S$ from the value given by Eq. (18.2) to $c$. Therefore the time required by a pointlike flash of light to cover the distance $\ell$ in a direction forming an angle $\theta$ with respect to the $x$ axis has to be modified in the following way
Given the homogeneity of space we assume, without loss of generality, that the flash of light is emitted in the origin, so that \( \ell \cos \theta = x \). Therefore the recipe is very simple: subtract \( x v/c^2 \) to the time shown by the clock having a position with first coordinate \( x \). Thus the new time \( \tilde{t} \) that should replace \( t \) for the considered clock is

\[
\tilde{t} = t - x \frac{v}{c^2}
\]  

(18.6)

If this is done systematically for all clocks at rest in \( S \) a new situation is obtained in which the speed of light relative to \( S \) appears to be isotropic and equal to \( c \). Obviously, the described ROC is the only procedure allowing one to obtain isotropy in \( S \).

Our next task is to find out how ROC should be implemented in the other inertial frames in order to preserve the validity of absolute simultaneity. According to Mansouri and Sexl [18-4] the absolute simultaneity can be obtained as follows: "... simply by choosing one system to be the ether system, synchronizing clocks according to the Einstein procedure in this system, and then synchronizing clocks in all other systems moving past \( \Sigma \) by adjusting these clocks to \( t = 0 \) whenever they fly past a clock in \( \Sigma \) which shows \( T = 0 \)."

[N.B. Here \( \Sigma \) is our \( S \), \( t \) has the same meaning as ours, and \( T \) is our \( t' \)]. The present chapter can be considered as providing a concrete implementation of the Mansouri-Sexl idea.

We start from \( S_0 \), the frame considered privileged before ROC. Given the point \( P \) fixed in \( S \) with coordinate \( x \), according to (18.6) in \( P \) we have \( \tilde{t} = 0 \) at the pre-ROC time

\[
t = x \frac{v}{c^2}
\]

(18.7)

At this time another point \( Q \), fixed in \( S_0 \) but mobile in \( S \), will overlap \( P \). Let \( x_0 \) be the \( Q \) coordinate in \( S_0 \) and \( x_Q \) the \( Q \) coordinate in \( S \) at \( t = 0 \). The (absolute) Lorentz contraction, typical of the inertial transformations, implies
that a segment of length $x_0$ at rest in $S_0$ is seen longer in $S$, namely that $x_0$ and $x_Q$ before ROC satisfy

$$x_Q R = x_0$$  \hspace{1cm} (18.8)

The point $Q$ had the pre-ROC velocity $-v/R^2$ relative to $S$, with which it covers the distance $QP$ from $t=0$ to the time $t$ satisfying (18.7). The situation is represented in Fig. 24 from the point of view of $S$. Therefore

$$-R^2 \frac{x - x_Q}{v} = x \frac{v}{c^2}$$  \hspace{1cm} (18.9)

whence, recalling the definition (6.3) of $R$

$$x_Q R^2 = x$$  \hspace{1cm} (18.10)

Eq.s (18.8) and (18.10) together give

$$x = Rx_0$$  \hspace{1cm} (18.11)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure24.png}
\caption{The inertial system $S_0$ is seen moving from the system $S$. The two points $P$ (fixed in $S$) and $Q$ (fixed in $S_0$) will coincide at some future time.}
\end{figure}
Comparing (18.11) with the pre-ROC inertial transformation of time \( t = t_0R \) one sees that the two pairs of variables \((x, t)\) and \((x_0, t_0)\) have the same ratio. It is then easy to show that at the time of \(QP\) overlapping one must have

\[
t_0 = x_0 \frac{v}{c^2}
\]

(18.12)

Therefore, if we define the \(S_0\) post-ROC time \(\tilde{t}_0\) as follows

\[
\tilde{t}_0 = t_0 - x_0 \frac{v}{c^2}
\]

(18.13)

the \(QP\) overlapping takes place at \(\tilde{t} = \tilde{t}_0 = 0\). The previous argument can be repeated for an arbitrary point \(P\) and shows that the absolute simultaneity is preserved in \(S\) and in \(S_0\) if the ROC process is carried out by using the definitions (18.6) and (18.13). The ROC is unique, as there cannot be two different points \(Q\) overlapping with \(P\) at the same time \(t\).

After dealing with the post-ROC and pre-ROC privileged systems, our task is to find out how ROC should be in the most general inertial system in order to preserve the absolute simultaneity. One can show that the ROC in \(S'\) should satisfy the following rule:

\[
\tilde{t}' = t' - x' \frac{R^2}{1 - v'v/c^2} \frac{v}{c^2}
\]

(18.14)

Thus we obtain that the \(Q'P'\) overlapping takes place at \(\tilde{t}' = \tilde{t}_0 = 0\). The previous argument can be repeated for an arbitrary point \(P'\) and shows that the absolute simultaneity is preserved if the ROC process is carried out by using the definitions (18.6) and (18.14).

We have found how ROC should be implemented in all inertial systems if the privileged one, \(S_0\), is to be replaced by \(S\) and absolute simultaneity is to be preserved. The most general recipe for ROC is given by Eq. (18.14) since Eq.s (18.6) and (18.13) are particular cases of (18.14) obtainable for \(v' = 0\) and \(v' = v\), respectively.

It can be shown that the ROC given by (18.14) maintains the validity of the inertial transformations with the new inertial system \(S\) replacing \(S_0\) in the
role of "privileged" system, while $S_0$ becomes a regular "moving" inertial system also from the point of view of the new transformations. Furthermore any other system $S'$ obtains transformations appropriate to the new roles of $S$ and $S_0$.

The research is concluded by considering a third system $S'$ which has no privilege either before or after ROC and establishing all the rules it has to satisfy. We omit this longer part which presents nothing conceptually new with respect to what we have seen. The details are in ref. [18-1]. In this way it remains established that ROC is possible, meaning that one can choose freely the rest system which is privileged and keep the full validity of the inertial transformations.

As repeatedly stated above, the adoption of the inertial transformations implies essentially a violation of Einstein’s relativity principle. Clearly the point needs to be discussed further.

It is useful to distinguish two formulations of the relativity principle:

R1. **TSRong relativity**, according to which the laws of physics are exactly the same in all inertial systems. This is Einstein’s formulation.

R2. **Weak relativity**, stating merely the impossibility to measure the absolute velocity of the Earth. This principle does not demand necessarily the validity of the Lorentz transformations and opens a logical space for new theories, such as the one based on the inertial transformations. This formulation is essentially the original one given by Galilei extended, of course, to all physical phenomena.

Experimental evidence based on the clock paradox (§ 14) and on stellar aberration (§ 13) shows that absolute velocities exist in nature. The weak relativity principle accepts this, but maintains that they nevertheless remain unmeasurable. The results of the present paper are in agreement with some of those obtained by Rizzi, Ruggiero and Serafini, who wrote: "… the 'privileged role' played by $S_0$ … is a merely artificial element, $S_0$ being just the IRF [inertial reference frame] in which, by stipulation, Einstein synchronization has been performed: as a matter of fact, any IRF $S$ can play the role of $S_0$." [18-6]

In spite of this conclusion, with which I agree, I must insist that the statements made in the initial part of the present book are correct. The theory with the free $e_1$ applied to the rotating platform and to the Sagnac effect shows
that only \( e_1 = 0 \) gives a rationally acceptable formulation of the physics on the disk. Similarly, only with \( e_1 = 0 \) one can obtain a reasonable description of aberration. The paradoxes of the special theory of relativity disappear if \( e_1 = 0 \). The growing evidence for the existence in nature of superluminal signals can easily be accommodated if \( e_1 = 0 \), while it is incompatible with standard relativity due to the presence of a famous causal paradox. Therefore the best theory of the physics of space and time seems to be the one based on absolute simultaneity.

Nevertheless we must admit that our results may seem somewhat contradictory. On the one hand they point to a theory of space and time in which such conceptions as absolute velocity, privileged frame and absolute simultaneity have a central role, while, on the other hand, relativism comes back in the arbitrariness of the choice of the "privileged" inertial reference frame. In spite of the fact that our results are mixed, we must stress that both sides of the contradiction (namely, \( e_1 = 0 \) and weak relativity) are absolutely correct if the assumptions made in the first section are correct. I can add that from the point of view of the inertial transformations the validity of weak relativity appears accidental, more than fundamental. It would be enough to discover a very small noninvariance of the two way speed of light to make the whole game of ROC impossible.

In the previous sections we saw that many paradoxes of relativity melt away rather easily if we adopt a more optimistic philosophy about our possibility to understand nature correctly. Furthermore, this "more optimistic" philosophy is not anymore a free choice, but has become a scientific necessity as a consequence of the six independent proofs of absolute simultaneity given above. Several paradoxes of the TSR were recalled in section 2. We will now shortly present the corresponding solutions by following the same order.

P1. The (relativistic) idea that the simultaneity of spatially separated events does not exist in nature and must therefore be established with a human choice was accepted by Mansouri and Sexl, who fully believed in the conventionality of clock synchronization. In spite of the broad diffusion of this type of expectation, in the present book it has been established that a rational description of physical phenomena (Sagnac effect, linearly accelerating systems, objective reality of inertial observers, superluminal propagations) can be obtained only if absolute simultaneity is adopted: \( e_1 = 0 \).
P2. The relativity of simultaneity, according to which two events simultaneous for an observer in general are no more such for a different observer was overcome in section 14, where it was shown that assuming $e_1=0$ all inertial observers have the same reality, where reality is defined by the set of events simultaneous with a given event (e.g., the "here-now" event establishing the local present).

P3. The velocity of a light signal, considered equal for observers at rest and observers pursuing it with velocities as near as possible to $c$. The answer of a theory based on the ITs is as follows. After having established that $e_1 = 0$, the velocity of light relative to a moving reference frame is given by Eq. (18.2). Therefore, the speed of the light signal (absolute velocity $c$) relative to an inertial frame which is running after it (then, $\theta = 0$) with absolute velocity almost equal to that of light ($v \approx c$) has a denominator $1+1=2$ and the limit velocity is $c/2$. This is a 50% reduction with respect to the relativistic prediction which remains the old dear $c$.

P4. The retardation of moving clocks, phenomenon for which the theory of relativity does not provide a description in terms of objectivity. As we discussed at length in chapter 14, the objectivity is restored with the inertial transformations in terms of action of the ether on all the periodic phenomena which can be used to measure time. All this is very much in the realistic line of thought of Hendrik Lorentz.

P5. The contraction of moving objects, phenomenon for which, once more, the theory does not provide a causal description. The objectivity is restored with the inertial transformations in terms of action of the ether on every atom, with reduction of the atomic length in the direction of motion. Also this follows the realistic line of thought of Lorentz.

P6. The hyperdeterministic block universe of relativity, fixing in the least detail the future of every observer, is now out, having been overcome thanks to the conclusion that $e_1 = 0$, meaning that the reality line introduced in section 12 is unique for all observers, independently of their state of motion.

P7. The conflict between the reciprocal transformability of mass and energy and the ideology of relativism. In section 3 we showed that the TSR declares all inertial observers perfectly equivalent so depriving energy of its full reality. The retrieval of the objectivity of energy and of the other physical
quantities should rather aim at the inequivalence of the different reference frames knowing that there is one at rest in the ether, which has a more fundamental role. The idea is developed in section 3, where the objectivity of energy is fully recovered by working with the inertial transformations.

P8. The TSR predicts a discontinuity between the inertial systems and systems endowed with a very small acceleration. The discontinuity is in the variable $\rho$, ratio of the velocities of light along two opposite directions. It turns out to be a very serious problem for all $e_1 \neq 0$. If one takes $e_1 = 0$, however, the discontinuity does not exist anymore and the relative difficulty is completely overcome.

P9. The propagations from the future towards the past, generated in the TSR by the eventual existence of superluminal signals. In section 17 we showed that the essence of the causal paradox lies in the impossible requirement that a superluminal propagation may overtake a set of clocks marking a progressively decreasing physical (then, not conventional) time. The particular choice $e_1 = 0$, which, as we saw, is selected by several phenomena in which the acceleration has a role, remains by far the best one also in the present context, being the only one not leading to the causal paradox. The same choice avoids the complications of the TSR describing all the propagations as forwards in time for all observers.

P10. The asymmetrical ageing of the twins in relative motion in a theory waving the flag of relativism. In section 14 we discussed the differential retardation effect between separating and reuniting clocks (“clock paradox”). A variational method was used to show, both in the TSR and in more general theories with arbitrary $e_1$, that among all possible trajectories of a clock connecting two given points at two given times the rectilinear uniform motion requires the longest proper time. A complete resolution of the clock paradox is so obtained by giving an exhaustive unified description of all possible situations. Relativism does not apply and must be considered obsolete. Velocity (and nothing else) is seen to be responsible for the differential retardation effect. Of course it must be an absolute velocity! Hidden behind the relativism of Einstein's theory there is a physically active background.

Tom Phipps has observed that the relativistic theory lacks the "robustness" that normally would be required in a physical theory: "The special
theory is a very special theory indeed. It has a feeling about it rather different from anything else that has appeared in physical science in the modern era. It is somehow more refined, more delicate, more finely-tuned. Like an over-bred race horse, it seems apt to break a leg in the first chuck-hole if let into in ordinary, real-world pasture." [18-7]

In fact, this is due to what can also be called the mathematical instability of the theory. To understand this point it is important to stress that the Lorentz transformations are necessary consequences of the relativity principle. In presenting the TSR one can always choose two Cartesian coordinate systems in the inertial reference frames \( S \) and \( S_0 \) by assuming:

1. that space is homogeneous and isotropic, and that time is homogeneous;
2. that in \( S_0 \) the velocity of light is the same in all directions, so that Einstein's synchronization can be applied and velocities relative to \( S_0 \) can be measured; that the origin of \( S \) (equation \( x = 0 \)) is seen from \( S_0 \) moving with velocity \( v \) parallel to the \(+x_0\) axis with equation of motion \( x_0 = vt_0 \);
3. that the observer in \( S \) sees his origin \((x = y = z = 0)\) coincident with that of \( S_0 \) at \( t = 0 \), and \textit{vice-versa} that the observer in \( S_0 \) sees his origin \((x_0 = y_0 = z_0 = 0)\) coincident with that of \( S \) at \( t_0 = 0 \);
4. that planes \((x_0, y_0)\) and \((x, y)\) coincide at all times \(t_0\); that also planes \((x_0, z_0)\) and \((x, z)\) coincide at all times \(t_0\);
5. that planes \((y_0, z_0)\) and \((y, z)\) coincide \textit{at time} \(t_0 = t = 0\).

It was shown in Ref. [18-8] that the previous conditions reduce necessarily the transformation laws from \( S_0 \) to \( S \) to the form

\[
\begin{align*}
x &= f_1(x_0 - vt_0) \\
y &= g_2 y_0 \\
z &= g_2 z_0 \\
t &= e_1 x_0 + e_4 t_0
\end{align*}
\]
where the four coefficients $f_1$, $g_2$, $e_4$, and $e_1$ can depend on $v$.

If at this point one assumes the validity of the relativity principle (and the invariance of the velocity of light) the previous transformations reduce necessarily to the Lorentz transformations.

It can therefore be concluded that any eventual violation of the Lorentz transformations found at any time in the future will imply that relativity itself does not hold as a description of nature. If one considers a four-dimensional space in which the four coefficients $f_1$, $g_2$, $e_4$, and $e_1$ are represented as Cartesian coordinates, one can say that for a given absolute velocity of the inertial frame $S$ all the coefficients are completely fixed, and therefore represented by a geometrical point. In this space there is no finite area describing relativity, only a structureless, unprotected point. All other points lead to the logical negation of the relativity principle.

One can say that the Special Theory of Relativity (TSR) is mathematically "unstable", in the sense that any shift, however small, of any one of the four coefficients $f_1$, $g_2$, $e_4$, and $e_1$ away from its relativistic value implies necessarily the existence of a privileged frame. In other words, either Lorentz has given mankind a final truth with his transformations, or the existence of a privileged frame shall be accepted in the future.

Phipps comments: "The very idea of "progress" is laughable in the context of special relativity theory. Being so delicately-tuned, it lacks the robustness that would allow minor amendments consistent with "progress"." By contrast "Galilean kinematics exemplifies a robust theory. Galilean inertial motions (those ol' Newton's mechanics) are not tied to perfection: Small departures invariably produce small effects. In hindsight one can perceive that this robustness derives from the affine nature of the classical geometry; that is, from the failure to assert any rigid connectedness or symmetry between "space" and "time"." [18-7]


Bibliography

[QP]  M. Ferrero & A. van der Merwe, eds., NEW DEVELOPMENTS ON


